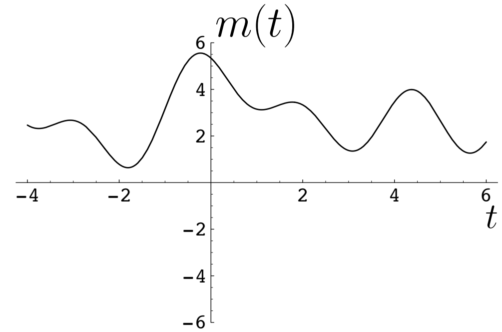
24 Signal transmission, circular polarization

Since in perfect dielectrics the propagation velocity $v_p = v$ and the intrinsic impedance η are frequency independent (i.e., propagation is non-dispersive), d'Alembert plane wave solutions of the form

$$\mathbf{E} = \hat{x} f(t - \frac{z}{v}) \quad \text{and} \quad \mathbf{H} = \hat{y} \frac{f(t - \frac{z}{v})}{\eta}$$

are valid in such media.

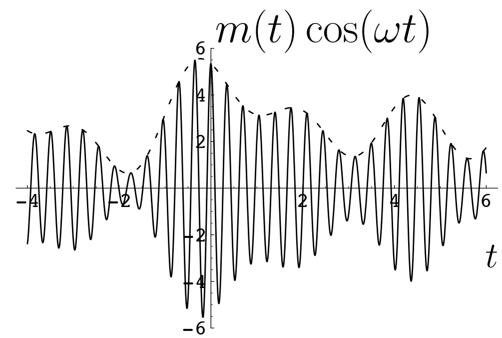


- Consider a waveform

$$f(t) = m(t)\cos(\omega t),$$

where

  - ω is some specific frequency having a corresponding period $T = \frac{2\pi}{\omega}$,
  - $m(t)$ is some arbitrary signal (e.g., a voice signal, a message) changing slowly compared to period T.



In that case,

  - $f(t)$ specified above can be called **narrowband AM**, and
  - ω the **carrier frequency** of modulating cosine of the message signal $m(t)$.

The corresponding x-polarized wave fields propagating in z direction can then be represented as **Field 1**

$$\mathbf{E} = m(t - \frac{z}{v})\cos(\omega t - \beta z)\hat{x} \quad \text{and} \quad \mathbf{H} = \frac{m(t - \frac{z}{v})}{\eta}\cos(\omega t - \beta z)\hat{y}$$

where $\beta = \omega\sqrt{\mu\epsilon}$ as usual[1].

- With reference to the expressions above, we could say that the AM wave field has an **x-polarized carrier**.

- By contrast, **Field 2**

$$\mathbf{E} = m(t - \frac{z}{v})\cos(\omega t - \beta z)\hat{y}$$

represents an AM wave field with a **y-polarized carrier**, and so does **Field 3**

$$\mathbf{E} = m(t - \frac{z}{v})\sin(\omega t - \beta z)\hat{y}$$

but with a carrier that has been time-delayed by a quarter period.

- Suppose Fields 1 and 3 above were transmitted simultaneously and therefore superpose. In that case we will have a wave field with **Circular polarized carrier**

$$\mathbf{E} = m(t - \frac{z}{v})[\cos(\omega t - \beta z)\hat{x} + \sin(\omega t - \beta z)\hat{y}]$$

[1]In ***dispersive*** media where β is a non-linear function of ω, narrowband AM can propagate as

$$m(t - \frac{z}{v_g})\cos(\omega t - \beta z)\hat{x} \quad \text{where} \quad v_g \equiv \frac{\partial\omega}{\partial\beta}$$

is known as **group velocity** — covered in detail in ECE 450.

which has a **circular polarized carrier**. Since this is just a superposition of two d'Alembert waves, the accompanying $\mathbf{H}$ is easily found to be

$$\mathbf{H} = m(t - \frac{z}{v})[\cos(\omega t - \beta z)\hat{y} - \sin(\omega t - \beta z)\hat{x}]/\eta.$$

– Circular-polarized AM wave fields just introduced are in some practical applications better to use than the linear-polarized waves because of, say, the peculiarities of a propagation medium (e.g, Earth's ionosphere or the interplanetary medium).

– Since a circular-polarized wave field is a linear combination of linear-polarized waves, it has a phasor that is a linear combination of phasors of its linear components, as in

$$\cos(\omega t - \beta z)\hat{x} + \sin(\omega t - \beta z)\hat{y} \;\Leftrightarrow\; e^{-j\beta z}\hat{x} - je^{-j\beta z}\hat{y} = (\hat{x} - j\hat{y})e^{-j\beta z}$$

or

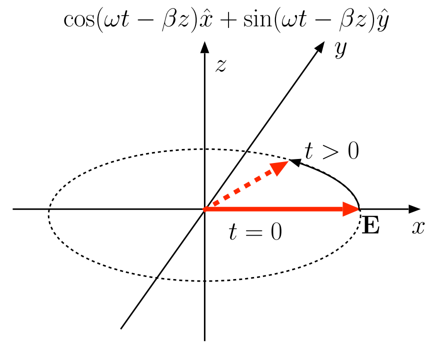
$$\cos(\omega t - \beta z)\hat{x} - \sin(\omega t - \beta z)\hat{y} \;\Leftrightarrow\; e^{-j\beta z}\hat{x} + je^{-j\beta z}\hat{y} = (\hat{x} + j\hat{y})e^{-j\beta z}.$$

- In the last step above, we have introduced two flavors of circularly polarized waves, which correspond to fields vectors rotating in opposite directions at any position in space when viewed toward the direction the wave propagates— clockwise for right-circular, counter-clockwise for left circular.

CIRCULAR POLARIZATION:

Field vector rotates instead of oscillating.

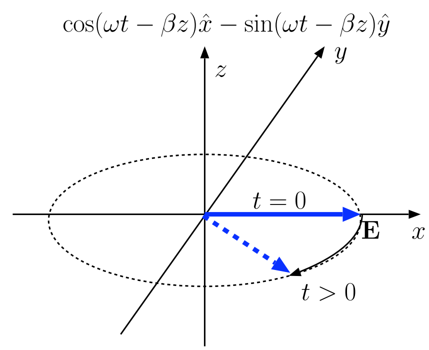
The rotation frequency is also the wave frequency.



RIGHT CIRCULAR

Right-circular

Left-circular



LEFT CIRCULAR

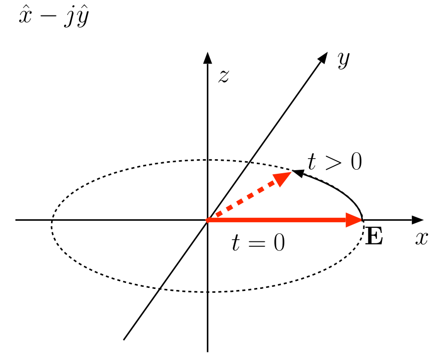
When left-hand thumb is pointed along propagation direction z the fingers curl in the rotation direction of the field vector.

- Also,

  - for the **right-circular** wave propagating in z direction, the field vector simplified at $z = 0$ as

    $$\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y} \quad \Leftrightarrow \quad \hat{x} - j\hat{y}$$

    rotates in the direction that your **right-hand** fingers curl when the thumb is directed in propagation direction z, whereas



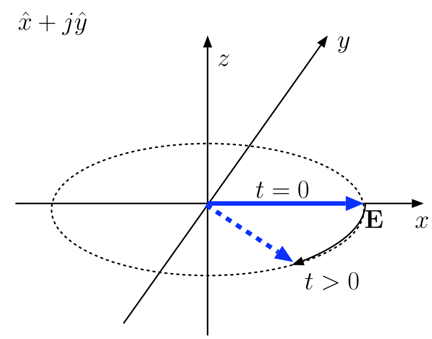
RIGHT CIRCULAR

x-comp leads y-comp because of -j

  - for the **left-circular** wave propagating in z direction, likewise, vector

    $$\cos(\omega t)\hat{x} - \sin(\omega t)\hat{y} \quad \Leftrightarrow \quad \hat{x} + j\hat{y}$$

    rotates in the direction that your **left-hand** fingers curl when the thumb is directed in propagation direction z.



LEFT CIRCULAR

x-comp lags y-comp because of +j

In general, the "handednes" or "helicity" of a circular polarized wave is always obtained by matching your *right* or *left* hand to the specified propagation and rotation directions — see example below.

Furthermore, the rotation direction is most easily seen if the wave is expressed in phasor form by seeing which component leads (or lags) which. Here is an explanation by example:

Example 1: A circular polarized wave field vector is given as

$$\tilde{\mathbf{E}} = (\hat{z} + j\hat{y})e^{j\beta x}.$$

Determine the propagation and rotation directions of the field vector as well as its helicity.

Solution:

The propagation direction is $-x$ since the exponent in $e^{j\beta x}$ lacks a minus sign.

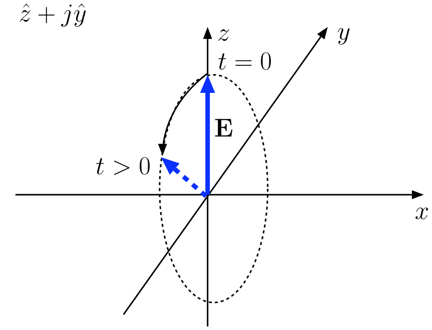
At $x = 0$, the wave field vector rotates as

$$\mathbf{E} = \mathrm{Re}\{(\hat{z} + j\hat{y})e^{j\omega t}\} = \hat{z}\cos(\omega t) - \hat{y}\sin(\omega t),$$

of which the y-component leads the z-component by $90°$ of phase, or, equivalently, by a quarter period in time — therefore, the vector points in y-direction before it points in z-direction (or in z-direction before it points in $-y$-direction), rotating from y- toward z-axis.

When I direct my right thumb in $-x$ direction, my fingers curl from z- toward y-axis, which is curling in the wrong direction. Hence this wave is not right-circular! It is **left-circular**.

$\hat{z} + j\hat{y}$



LEFT CIRCULAR

(i) z-comp lags y-comp because of +j.

(ii)E-vector rotates from y towards z.

(iii) since the wave propagates in -x direction it is LEFT CIRCULAR

Given any propagation direction, a carrier field of an arbitrary polarization can always be expressed as weighted superpositions of any pair of **orthogonal polarized** carrier fields — such orthogonal pairs are considered to be complete sets of **basis functions** for expressing waves with arbitrary polarizations.

Circulars in terms of linears

- **EXAMPLE:** Right- and left circular waves propagating in z directions are weighted superpositions of **orthogonal x- and y-polarized** fields as in (expressed in terms of phasors): basis functions

  $$\hat{x}e^{-j\beta z} \text{ and } \hat{y}e^{-j\beta z}$$

  superpose to form right- and left-circular waves

  $$(\hat{x} - j\hat{y})e^{-j\beta z} \text{ and } (\hat{x} + j\hat{y})e^{-j\beta z}$$

  using the weights

  $$1,\ -j \text{ and } 1,\ j$$

  respectively.

Linears in terms of circulars

- **EXAMPLE:** x- and y-polarized waves propagating in z directions are weighted superpositions of **orthogonal right- and left-circular** fields as in (expressed in terms of phasors): basis functions

  $$(\hat{x} - j\hat{y})e^{-j\beta z} \text{ and } (\hat{x} + j\hat{y})e^{-j\beta z}$$

  superpose to form linear polarized waves

  $$\hat{x}e^{-j\beta z} \text{ and } \hat{y}e^{-j\beta z}$$

  using the weights

  $$\frac{1}{2}, \frac{1}{2} \text{ and } -\frac{1}{2j}, \frac{1}{2j}$$

  respectively.

- It can be argued that right- and left-circular wave pair forms an intrinsically more fundamental set of basis functions than, say, $\hat{x}$- and $\hat{y}$-polarized waves, because while the selection of which direction is x and which direction is y can be arbitrary, there is no arbitrariness in how helicity is assigned to circular polarized modes propagating in a given direction.

- Also, oscillating charges will radiate linear-polarized fields, whereas rotating charges will radiate circular-polarized fields (in the direction normal to the rotation plane) — so, source dynamics selects the radiated wave polarization.

- Wave polarization is important because

  - it depends on physical geometry and dynamics of the wave source,
  - it may depend on the physical properties of the region the wave propagates through,
  - it will determine the direction of Lorentz force on any test charge or electrical load,
  - angular momentum carried by the wave depends on polarization, etc.

Example 2: On $z = 0$ plane we have a time-varying surface current density

$$\mathbf{J}_s(t) = m(t)[\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}]\,\frac{\mathrm{A}}{\mathrm{m}}$$

with a carrier frequency of ω. Determine the radiated wave fields $\mathbf{E}^{\pm}$ and the polarization (and the helicity if appropriate) of the carrier.

Solution: We have already learned that a surface current $\mathbf{J}_s(t)$ on $z = 0$ plane will produce TEM wave fields

$$\mathbf{E}^{\pm} = -\frac{\eta}{2}\mathbf{J}_s(t \mp \frac{z}{v})$$

in surrounding regions. With the given $\mathbf{J}_s(t)$, this implies

$$\mathbf{E}^{\pm} = -\frac{\eta}{2}m(t \mp \frac{z}{v})[\cos(\omega t \mp \beta z)\hat{x} + \sin(\omega t \mp \beta z)\hat{y}]\,\frac{\mathrm{V}}{\mathrm{m}},$$

which has a circular-polarized carrier

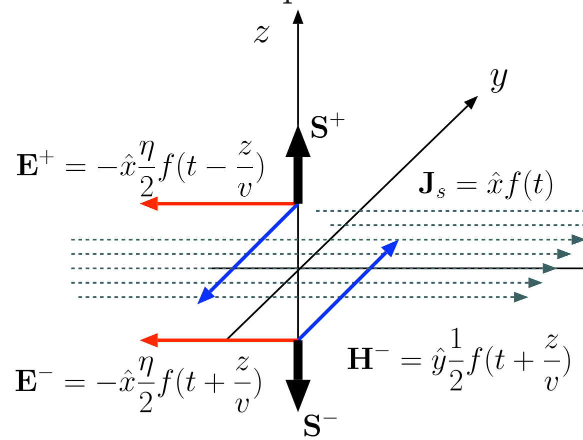
$$\cos(\omega t \mp \beta z)\hat{x} + \sin(\omega t \mp \beta z)\hat{y}$$

that varies, on $z = 0$ plane, as

$$\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}.$$

This vector rotates from x- toward y-axis, and therefore the carrier of $\mathbf{E}^+$ is right-circular and the carrier of $\mathbf{E}^-$ is left-circular.

Note that this figure only shows one linear component of the surface current on $z = 0$ plane. One linear component causes a linear polarized radiation. An orthogonal pair of linear components will conspire to radiate a circular polarized wave as in Example 2 when they are 90° out of phase.



Example 3: In Example 2, what is the average power density of the circular polarized carrier signal

$$\mathbf{E}_c = \cos(\omega t - \beta z)\hat{x} + \sin(\omega t - \beta z)\hat{y}\,\frac{\mathrm{V}}{\mathrm{m}}$$

in the region $z > 0$, assumed to be vacuum?

Solution: In phasor notation $\mathbf{E}_c$ and is given as

$$\tilde{\mathbf{E}}_c = (\hat{x} - j\hat{y})e^{-j\beta z}\,\frac{\mathrm{V}}{\mathrm{m}}.$$

The corresponding $\mathbf{H}_c$ phasor is

$$\tilde{\mathbf{H}}_c = \frac{1}{\eta_o}(\hat{y} + j\hat{x})e^{-j\beta z}\,\frac{\mathrm{V}}{\mathrm{m}}.$$

Therefore, the average power density is found to be

$$\frac{1}{2}\mathrm{Re}\{\tilde{\mathbf{E}}_c \times \tilde{\mathbf{H}}_c^*\} = \frac{1}{2\eta_o}\mathrm{Re}\{(\hat{x} - j\hat{y}) \times (\hat{y} + j\hat{x})^*\} = \frac{1}{2\eta_o}(\hat{z} + \hat{z}) = \frac{1}{\eta_o}\hat{z}.$$

This is twice the power content of a linearly polarized wave field of an equal amplitude!

Make sure you check and follow all the sign changes that take place in Example 3.