29 Bounce diagram examples

Example 1: A transmission line of length $\ell = 900$ m and speed $v = c$ has $R_L = 2Z_o$. It is excited by a generator having an open circuit voltage $f_i(t) = \sin(\omega t)u(t)$ and Thevenin resistance $R_g = Z_o$. The source frequency is $\frac{\omega}{2\pi} = 1\text{MHz}$. Determine the voltage response $V(z,t)$ in the circuit after first determining the impulse response function $h_z(t)$.

Solution: In the circuit, the injection coefficient is

$$\tau_g = \frac{Z_o}{R_g + Z_o} = \frac{Z_o}{Z_o + Z_o} = \frac{1}{2}.$$

The load and generator reflection coefficients are

$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} = \frac{2Z_o - Z_o}{2Z_o + Z_o} = \frac{2-1}{2+1} = \frac{1}{3} \text{ and } \Gamma_g = \frac{R_g - Z_o}{R_g + Z_o} = \frac{Z_o - Z_o}{Z_o + Z_o} = 0,$$

respectively. Also, time-delay

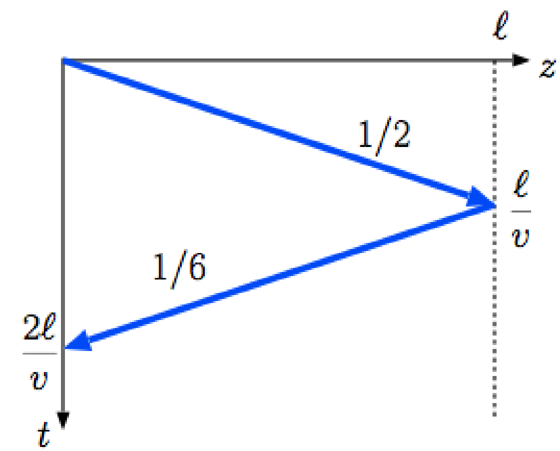
$$\frac{2\ell}{v} = \frac{2 \cdot 900\,\text{m}}{300\,\text{m}/\mu\text{s}} = 6\,\mu\text{s}.$$

The corresponding bounce diagram is shown in the margin. Note that the diagram terminates at $t = \frac{2\ell}{v}$ because Z_o is matched to R_g and $\Gamma_g = 0$. From the diagram we deduce the impulse response

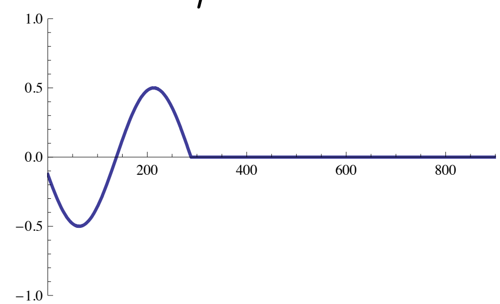
$$h_z(t) \;=\; \frac{1}{2}\delta(t - \frac{z}{c}) + \frac{1}{6}\delta(t + \frac{z}{v} - 6\mu).$$

Convolving $h_z(t)$ with $\sin(\omega t)u(t)$, we obtain the voltage response of the TL to the sinusoidal input as (see plots in the margin)

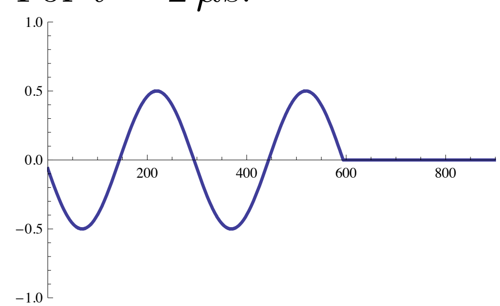
$$V(z,t) = h_z(t) * \sin(\omega t)u(t) = \frac{1}{2}\sin\omega(t - \frac{z}{v})u(t - \frac{z}{v}) + \frac{1}{6}\sin\omega(t + \frac{z}{v} - 6)u(t + \frac{z}{v} - 6).$$



For $t = 1\,\mu$s:



For $t = 2\,\mu$s:



See an animated version of this and another where $\Gamma_L = -1$

linked in the class calendar.

Example 2: Consider a TL circuit where $Z_o = 50\,\Omega$, $v = c$, $\ell = 2400$ m, $R_g = 0$, and $R_L = 100\,\Omega$. Determine and plot $V(1200, t)$ if $f_i(t) = u(t)$.

Solution: For this circuit

$$\tau_g = \frac{Z_o}{R_g + Z_o} = 1, \quad \Gamma_g = \frac{R_g - Z_o}{R_g + Z_o} = -1, \quad \text{and} \quad \Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} = \frac{1}{3}.$$

Also, the transit time across the TL is

$$\frac{\ell}{v} = \frac{2400\,\mathrm{m}}{300 \times 10^6\,\mathrm{m/s}} = 8\,\mu\mathrm{s}.$$

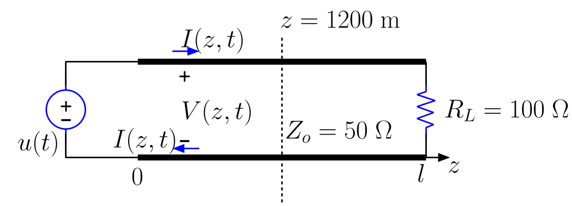
From the bounce diagram shown in the margin, the impulse response for $z = 1200$ m (the location marked by the vertical dashed line) is found to be

$$V(1200, t) = \delta(t-4) + \frac{1}{3}\delta(t-12) - \frac{1}{3}\delta(t-20) - \frac{1}{9}\delta(t-28) + \frac{1}{9}\delta(t-36) + \cdots$$

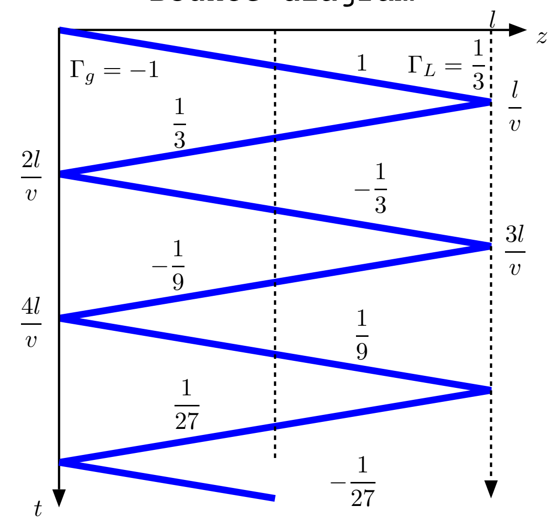
Replacing the $\delta(t)$ in this expression with the unit-step $u(t)$, the specified input signal $f_i(t)$, we get

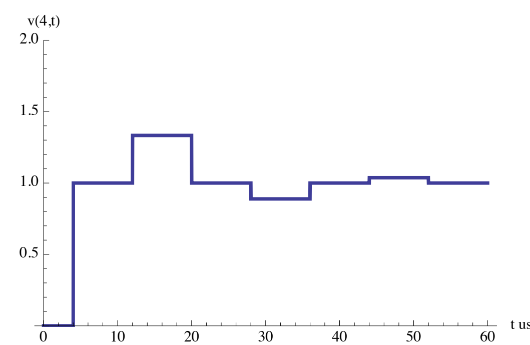
$$V(1200, t) = u(t-4) + \frac{1}{3}u(t-12) - \frac{1}{3}u(t-20) - \frac{1}{9}u(t-28) + \frac{1}{9}u(t-36) + \cdots$$

which is plotted in the margin.



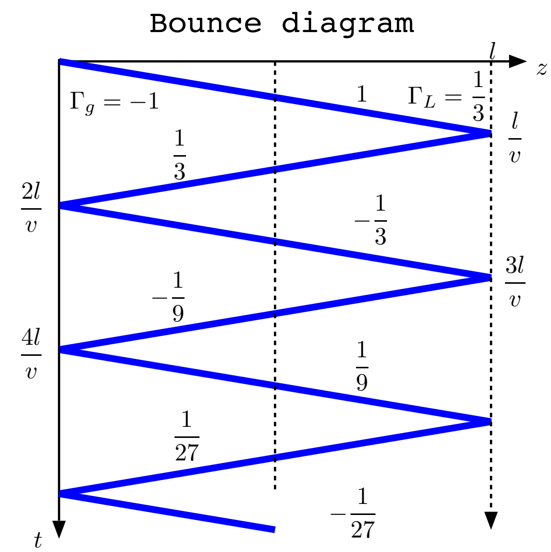
Bounce diagram

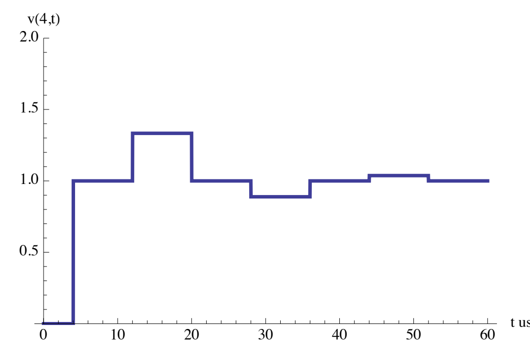




Animated version of this is linked in the class calendar.

- Note that as $t \to \infty$, $V(1200, t) \to 1$ V in Example 1, as if DC conditions prevail and the TL becomes a pair of wires in the lumped circuit sense.
    - DC steady-state corresponds to $\omega = 0$ and signal wavelength $\lambda \to \infty$. In that limit $\ell \ll \lambda$ is always valid and TL can be treated like an ordinary lumped circuit.
    - Of course this simplification can only occur with $f_i(t) = u(t)$, or its delayed/scaled versions, which are all asymptotically DC in $t \to \infty$ limit. The simplification does not apply for $f_i(t) = \sin(\omega t)u(t)$, for example.





Example 3: In the TL circuit described in Example 2, determine $V(z,t)$ and $I(z,t)$ for a new input signal $f_i(t) = \text{rect}(\frac{t}{T}) + 2\text{rect}(\frac{t-T}{T})$, $T = 1\ \mu$s. Plot $V(z,t)$ versus z at $t = 3\ \mu s$ and $t = 11\ \mu s$.

Solution: With $\tau_g = 1$, $\Gamma_g = -1$, $\Gamma_L = \frac{1}{3}$, and $\frac{2l}{c} = 16\,\mu$s, we obtain, by convolving with the general impulse response, the voltage response

$$V(z,t) = \sum_{n=0}^{\infty}(-\frac{1}{3})^n f_i(t - \frac{z}{c} - n16) + \frac{1}{3}\sum_{n=0}^{\infty}(-\frac{1}{3})^n f_i(t + \frac{z}{c} - (n+1)16)$$

where $\frac{z}{c}$ is to be entered in μs units. Also,

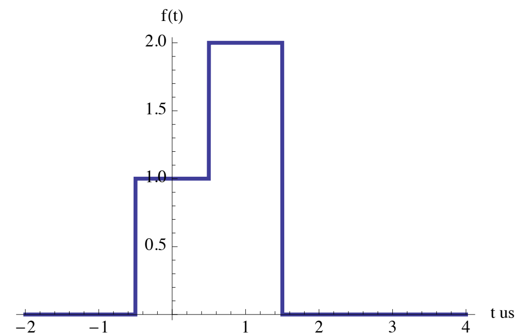
$$I(z,t) = \frac{1}{50}\sum_{n=0}^{\infty}(-\frac{1}{3})^n f_i(t - \frac{z}{c} - n16) - \frac{1}{50}\frac{1}{3}\sum_{n=0}^{\infty}(-\frac{1}{3})^n f_i(t + \frac{z}{c} - (n+1)16).$$

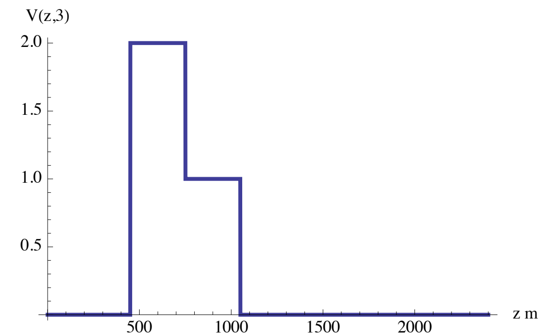
At $t = 3\ \mu s$, the voltage variation is

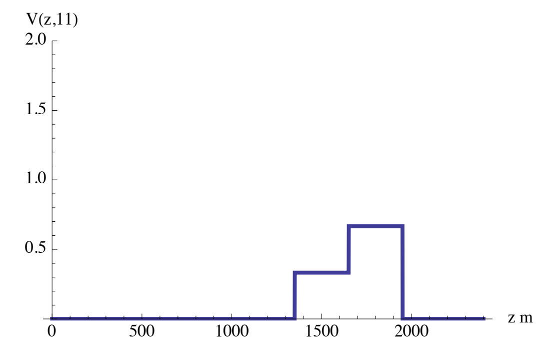
$$V(z,3) = \sum_{n=0}^{\infty}(-\frac{1}{3})^n f_i(3 - \frac{z}{c} - n16) + \frac{1}{3}\sum_{n=0}^{\infty}(-\frac{1}{3})^n f_i(3 + \frac{z}{c} - (n+1)16),$$

which is plotted in the margin using $f_i(t) = \text{rect}(t) + 2\text{rect}(t-1)$. Likewise, at $t = 11\ \mu s$,

$$V(z,11) = \sum_{n=0}^{\infty}(-\frac{1}{3})^n f_i(11 - \frac{z}{c} - n16) + \frac{1}{3}\sum_{n=0}^{\infty}(-\frac{1}{3})^n f_i(11 + \frac{z}{c} - (n+1)16).$$







Animated version of this is linked in the class calendar.