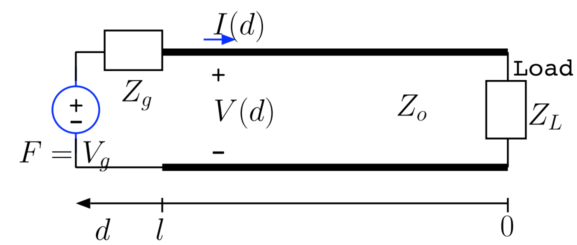
37 Smith Chart and impedance matching



- In lossless TL circuits the average power input P_{in} at the generator end precisely matches the average power delivered to the load, P_L.

  In fact, P_{in} and P_L also match the average power $P(d)$ transported on the line at an arbitrary d.

- We have in general

$$
\begin{aligned}
P(d) &= \frac{1}{2}\mathrm{Re}\{V(d)I^*(d)\} \\
&= \frac{1}{2}\mathrm{Re}\{(V^+e^{j\beta d} + V^-e^{-j\beta d})(\frac{V^+e^{j\beta d} - V^-e^{-j\beta d}}{Z_o})^*\} \\
&= \frac{1}{2}\mathrm{Re}\{\frac{|V^+|^2}{Z_o} - \frac{|V^-|^2}{Z_o} + \frac{V^-V^{+*}e^{-j2\beta d} - (V^-V^{+*}e^{-j2\beta d})^*}{Z_o}\} \\
&= \frac{|V^+|^2}{2Z_o} - \frac{|V^-|^2}{2Z_o}.
\end{aligned}
$$

  – Note that $P(d)$ is the difference of power transported
  $\frac{|V^+|^2}{2Z_o}$ **toward the load** by the "forward-going" wave, and
  $\frac{|V^-|^2}{2Z_o}$ **toward the generator** by the reflected wave.

  – Also note that

$$P(d) = \frac{|V^+|^2}{2Z_o} - \frac{|V^-|^2}{2Z_o} = \frac{|V^+|^2}{2Z_o}(1 - |\Gamma_L|^2)$$

  so that $|\Gamma_L|^2$ is an effective **power reflection coefficient**.

Power tx'ed toward the load:

$$\frac{|V^+|^2}{2Z_o}.$$

Power tx'ed toward the generator:

$$\frac{|V^-|^2}{2Z_o}.$$

Power reflection coefficient:

$$|\Gamma_L|^2.$$

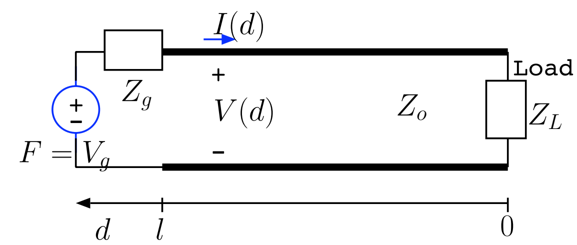
Power transmission coeff.:

$$1 - |\Gamma_L|^2.$$

- In TL circuits with load impedances Z_L **unmatched** to the characteristic impedance Z_o, the **reflected power**

$$\frac{|V^+|^2}{2Z_o}|\Gamma_L|^2$$

will be non-zero and the VSWR>1.



This a condition not favored by practical signal generators used in TL circuits.

- Most generators are *designed* (in their biasing arrangements) to operate in circuits with low VSWR (close to unity), requiring Z_{in} closely matched to R_g, most frequently $50\,\Omega$, an optimal characteristic impedance value for coax-lines (when line losses are taken into account).

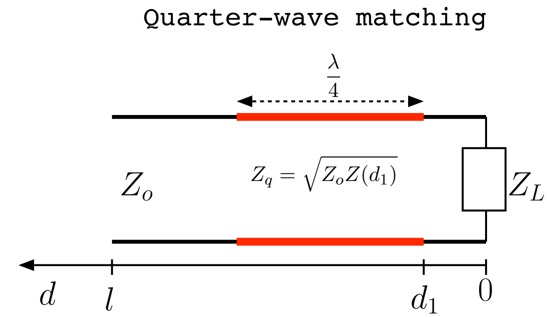
- Thus a standard procedure is to use TL's with $Z_o = R_g$, and utilize a *lossless* **impedance matching network** on the TL if the load impedance $Z_L \neq Z_o$.

  – This practice is called **impedance matching**.

Impedance matching achieves VSWR=1 between the generator and the matching network inserted at a location between the load and the generator.

- The inserted network should be designed to yield an input impedance equal Z_o at its input terminals.

The following examples illustrate different ways of achieving an impedance match.



Example 1: ***Quarter-wave matching* of resistive loads:**

Consider a TL with $Z_L = 25\,\Omega$ and $R_g = Z_o = 50\,\Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the VSWR>1.

To reduce the VSWR on the line connected to the generator to unity, we can insert a **quarter-wave transformer** right after Z_L — i.e., at $d_1 = 0$ in the circuit shown in the margin — with a characteristic impedance

$$Z_q = \sqrt{25 \times 50} = \sqrt{1250} = 35.35\,\Omega.$$

The impedance at the input terminals of the quarter-wave transformer (on the left) is then Z_o, i.e., 50 Ω, implying a perfect impedance match.

- Quarter-wave matching illustrated above is a very commonly used matching technique.
- It is a straightforward application of the quarter-wave transformer impedance formula
$$Z_{in} = \frac{Z_q^2}{Z_L}$$
for a transformer with characteristic impedance Z_q.

Example 2: ***Quarter-wave matching*** of reactive loads:

Consider a TL with $Z_L = 50 + j50\,\Omega$ and $R_g = Z_o = 50\,\Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the VSWR>1.

We cannot insert the quarter-wave transformer right after the load because then we would need a complex valued Z_q implying a lossy matching network.

Instead, we insert a **quarter wave transformer** a distance d_1 to the left of Z_L, where d_1 is selected, using a SC, to have a purely resistive $Z(d_1)$. In that case, the quarter-wave transformer impedance formula

$$Z_q = \sqrt{Z(d_1) \times 50}$$

yields a real valued Z_q as needed. This procedure leads to having $d_1 = d_{max}$ or $d_1 = d_{min}$ corresponding to the positions of voltage maxima and minima on the line.

As shown in the margin,

$$Z(d_1) = 50(2.62 + j0) = 131\,\Omega.$$

for

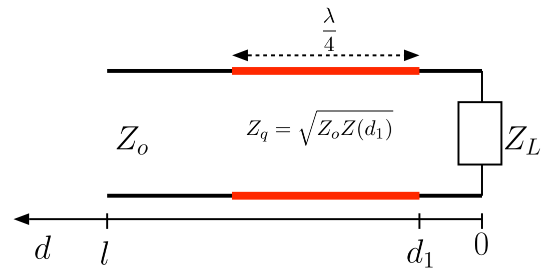
$$d_1 \approx 0.250\lambda - 0.162\lambda = 0.088\lambda$$

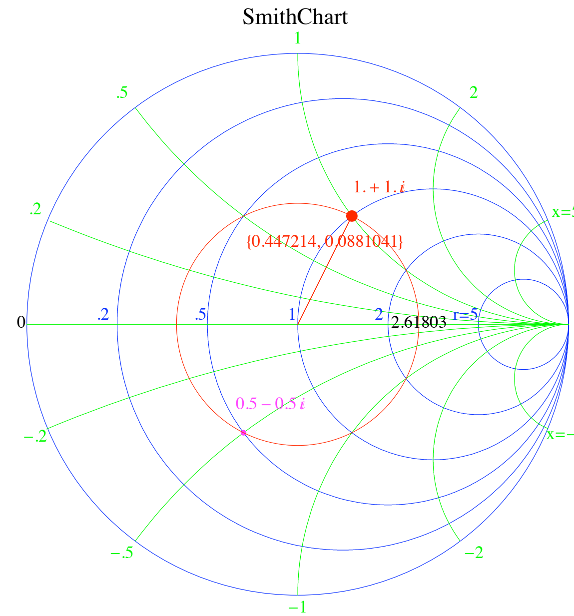
is a suitable choice for quarter-wave matching. In that case we need

$$Z_q = \sqrt{131 \times 50} = 50 \times \sqrt{2.62}\,\Omega$$

for the quarter wave transformer in order match to load to a line with $Z_o = 50\,\Omega$.

Quarter-wave matching





Note that:

$$z(d_1) = z(d_{max}) = \textbf{VSWR} \approx 2.62$$

as marked on the SC.
Also

$$d_{max} \approx 0.088\lambda$$

since, as marked on the SC, the angle of Γ_L is 0.088λ.

Example 3: ***Single-stub tuning:***

Consider a TL with $Z_L = 100 - j50\,\Omega$ and $R_g = Z_o = 50\,\Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the VSWR>1.

We will insert a **shorted-stub** a distance d_1 to the left of Z_L in parallel with the line to achieve an impedance match.

Distance d_1 will be selected, using a SC, to have a normalized admittance of

$$y(d_1) = 1 + jb$$

so that a stub, with a normalized input admittance

$$y_{stub} = -jb,$$

can be added in parallel to have a combined admittance of

$$y(d_1) + y_{stub} = 1 + j0$$

and achieve a perfect impedance match (i.e., VSWR=1).

In specific

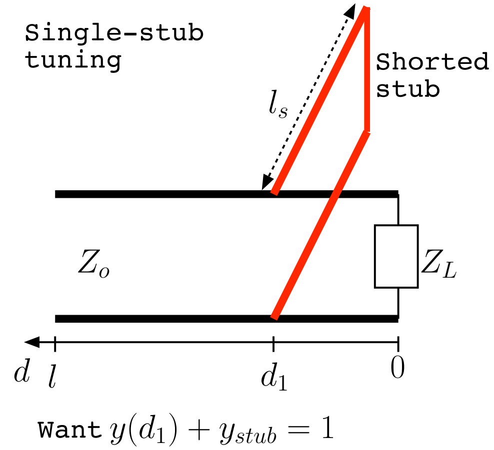
$$z_L = \frac{Z_L}{Z_o} = 2 - j1 \text{ and } y_L = \frac{1}{z_L} = 0.4 + j0.2$$

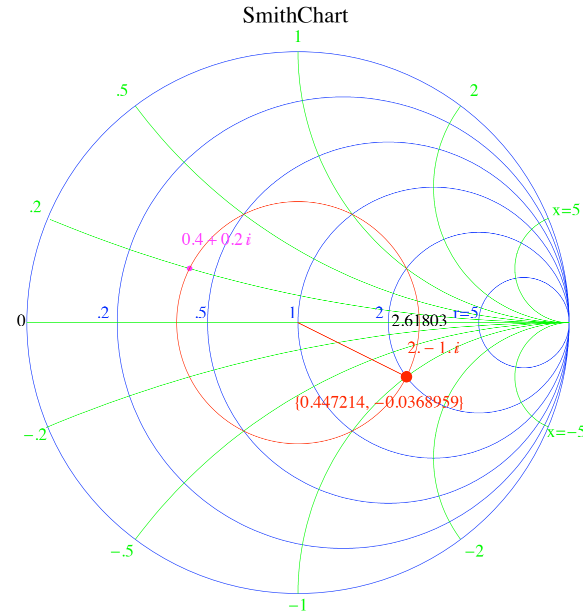
as shown on the SC on the top in the margin. We rotate clockwise on the SC by an amount corresponding to d_1 to obtain

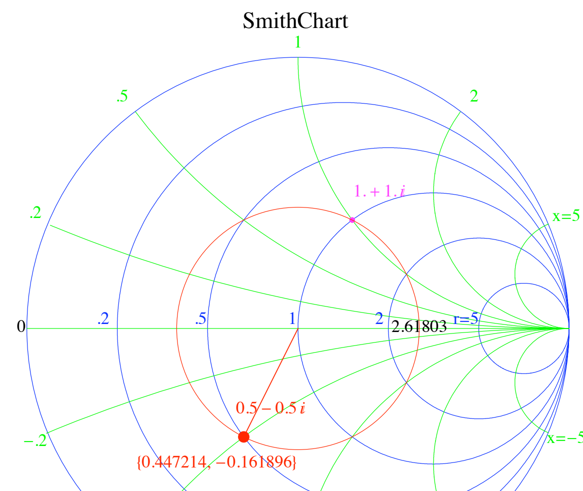
$$y(d_1) = 1 + j1$$

on the "$g = 1$" or "$y = 1 + jb$" circle as shown in the bottom SC. From the amount of rotation we determine

$$d_1 \approx 0.162\lambda - 0.037\lambda = 0.125\lambda.$$







The required input impedance of the shorted stub to achieve

$$y(d_1) + y_{stub} = 1 + j0$$

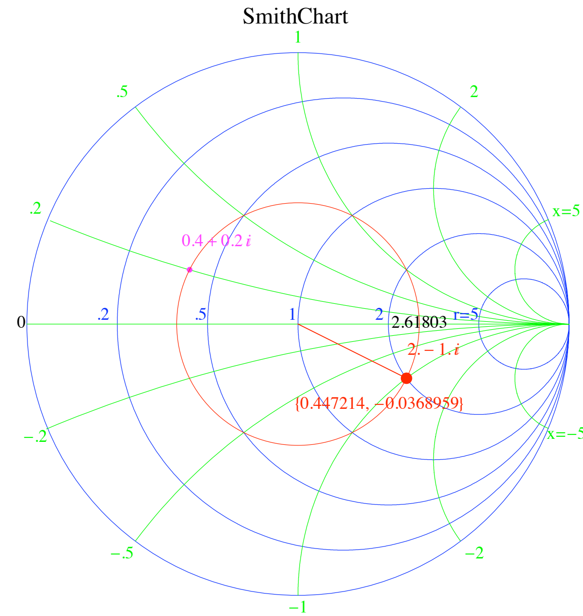
is

$$y_{stub} = -1j.$$

To achieve this input admittance the required stub length is

$$l_s = \frac{\lambda}{8} = 0.125\lambda$$

as determined from the SC — start at $y = \infty$ point on the SC on the far right (corresponding to the short termination), and then rotate clockwise (toward the generator) until the normalized admittance reads $-j1$; the amount of rotation indicates the required l_s.



- Another matching technique called ***double-stub tuning*** uses *two* shorted stubs of lengths l_1 and l_2 located at fixed values of d_1 and d_2.
  - Typically d_1 is zero or $\frac{\lambda}{4}$, and
  - $d_2 = d_1 + 3\frac{\lambda}{8}$.

    Vary l_1 and l_2 until VSWR is reduced to 1 near the generator end.

  The advantage of double-stub tuning is avoiding changes of stub locations when Z_L is changed. It's implementation on a SC is considerably more complicated than single-stub tuning.