8 Radiation fields of dipole antennas

- Radiation fields of a $\hat{z}$-directed Hertzian dipole are repeated in the margin.

- In this lecture we will first obtain the radiation fields of **short dipole** antennas by superposing the Hertzian dipole fields.

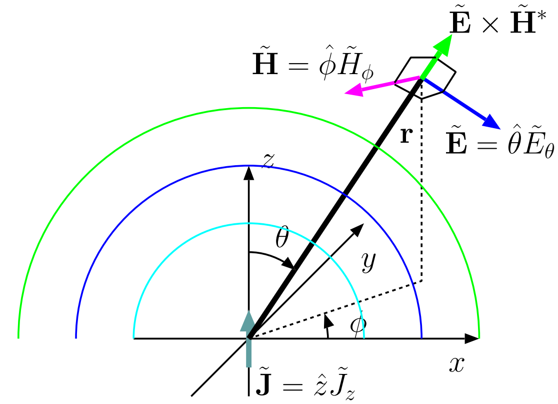
- A "short dipole" is a practical antenna — as opposed to a hypothetical Hertzian dipole — consisting of a pair of thin straight conducting wires of equal lengths $\frac{L}{2}$ placed along a common axis leaving a short gap between them (see margin).

  - A short dipole is typically used by connecting a "source" across the gap that constitutes the "input port" of the dipole antenna.
  - Let's assume that the source is an independent current source

    $$I(t) = I_o \cos \omega t \text{ A} \quad \Leftrightarrow \quad \tilde{I} = I_o \angle 0 = I_o \text{ A}$$

    and that the gap is an infinitesimal Δz so that the dipole and its input port occupy the region $-\frac{L}{2} < z < \frac{L}{2}$ in total.
  - We can then envision the entire dipole, including its input port, to be a stack of Hertzian dipoles of lengths Δz, with each Hertzian dipole centered about position z (in the interval $-\frac{L}{2} < z < \frac{L}{2}$) carrying a current $\tilde{I}(z)$, subject to boundary conditions

    $$\tilde{I}(0) = I_o \angle 0 \text{ A} \quad \text{and} \quad \tilde{I}(\pm\frac{L}{2}) = 0.$$

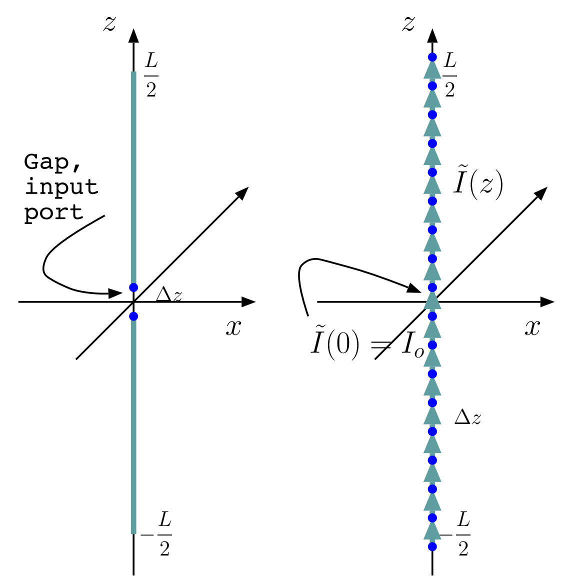


Radiation fields:

$$\tilde{\mathbf{E}} = j\eta_o I k \Delta z \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\theta}$$

and

$$\tilde{\mathbf{H}} = jIk\Delta z \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\phi}.$$

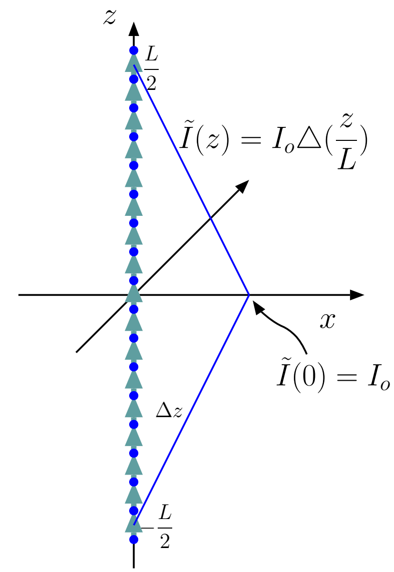


(a) Short dipole---undriven

(b) Driven short dipole modeled as a stack of Hertzian dipoles

In conformity with these boundary conditions we will assume that $\tilde{I}(z)$ is a *triangular* current distribution

$$\tilde{I}(z) = I_o \triangle(\frac{z}{L})\,\mathrm{A}.$$



Triangular current distribution

- What are the *radiation fields* of the short dipole antenna described above?

  We can answer this question in several different ways:

  1. We could turn the specified $\tilde{I}(z)$ into a corresponding $\tilde{\mathbf{J}}(z)$, and then, in succession, calculate the retarded potential $\tilde{\mathbf{A}}$, the magnetic field $\tilde{\mathbf{B}} = \nabla \times \tilde{\mathbf{A}}$, and then obtain $\tilde{\mathbf{E}}$ from $\tilde{\mathbf{B}}$ using Ampere's law as we did for the Hertzian dipole. Finally, the storage fields decaying faster with distance than $\frac{1}{r}$ would be dropped from $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ to obtain the radiation fields exclusively.
  2. A variant of (1), but with the radiation field $\tilde{\mathbf{H}}$ immediately deduced from $\tilde{\mathbf{B}}$, and then $\tilde{\mathbf{E}}$ is obtained by multiplying $\tilde{\mathbf{H}}$ by η_o and rotating it by 90° so that $\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*$ points in $\hat{r}$ direction.
  3. Superpose shifted and scaled versions of the radiation fields of the Hertzian dipole (as we will do shortly).

  All these options enumerated above will work because Maxwell's equations and radiation process have linearity properties.

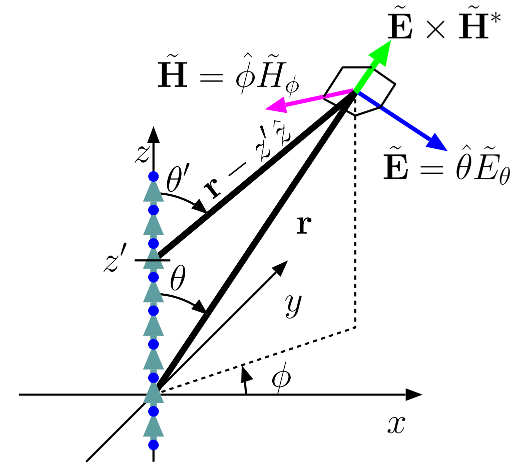
- The radiation electric field

$$\tilde{\mathbf{E}} = j\eta_o I k\Delta z \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\theta}$$

of a $\hat{z}$-directed Hertzian dipole

$$\tilde{\mathbf{J}} = \hat{z} I\Delta z \delta(x)\delta(y)\delta(z)$$

implies the following linear relationships for a $\hat{z}$-polarized radiation process, where the input function shown on the left represents the current distribution of the radiator:



Field due to dipole at the origin

$$\tilde{\mathbf{E}} = j\eta_o \tilde{I}(0) k\Delta z \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\theta}$$

Field due to displaced dipole

$$\tilde{\mathbf{E}} = j\eta_o \tilde{I}(z') k\Delta z \sin\theta' \frac{e^{-jk|\mathbf{r}-z'\hat{z}|}}{4\pi|\mathbf{r}-z'\hat{z}|}\hat{\theta}'$$

$$\delta(z) \rightarrow \boxed{\hat{z}\text{-pol radiator}} \rightarrow j\eta_o k \sin\theta \frac{e^{-jk|\mathbf{r}|}}{4\pi|\mathbf{r}|}\hat{\theta}$$

and

$$\delta(z-z') \rightarrow \boxed{\hat{z}\text{-pol radiator}} \rightarrow j\eta_o k \sin\theta' \frac{e^{-jk|\mathbf{r}-z'\hat{z}|}}{4\pi|\mathbf{r}-z'\hat{z}|}\hat{\theta}',$$

where (see margin)

$$\cos\theta' = \hat{z} \cdot \frac{\mathbf{r}-z'\hat{z}}{|\mathbf{r}-z'\hat{z}|};$$

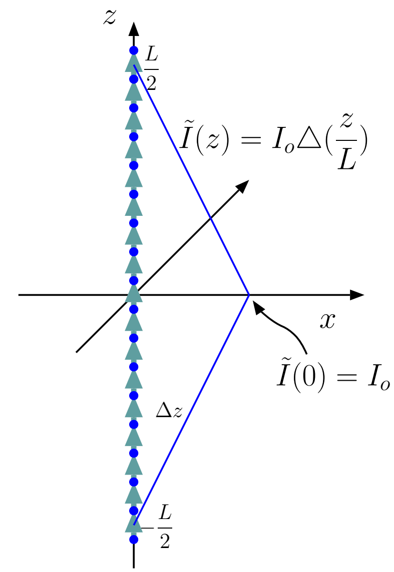
the implication is then

$$\int \tilde{I}(z')\delta(z-z')dz' = \tilde{I}(z) \rightarrow \boxed{\hat{z}\text{-pol radiator}} \rightarrow \int j\eta_o \tilde{I}(z') k \sin\theta' \frac{e^{-jk|\mathbf{r}-z'\hat{z}|}}{4\pi|\mathbf{r}-z'\hat{z}|}\hat{\theta}' dz' = \tilde{E}(\mathbf{r})$$

- The final result, the expression

$$\tilde{E}(\mathbf{r}) = \int j\eta_o \tilde{I}(z')k\sin\theta' \frac{e^{-jk|\mathbf{r}-z'\hat{z}|}}{4\pi|\mathbf{r}-z'\hat{z}|}\hat{\theta}'dz'$$

for the radiation electric field phasor is in fact very general, and applicable to dipole antennas of *all lengths* provided that the current distribution $\tilde{I}(z)$ on the dipole is known.



Triangular current distribution

- In practice, the triangular current distribution

$$\tilde{I}(z) = I_o\triangle(\frac{z}{L})\ \mathrm{A}$$

we have described earlier turns out to be applicable only when the dipole length

$$L \ll \lambda = \frac{c}{f}$$

at the operation frequency $f = \frac{\omega}{2\pi}$. For such dipoles

$$\begin{aligned}
\tilde{E}(\mathbf{r}) &= \int j\eta_o I_o \triangle(\frac{z'}{L})k\sin\theta' \frac{e^{-jk|\mathbf{r}-z'\hat{z}|}}{4\pi|\mathbf{r}-z'\hat{z}|}\hat{\theta}'dz' \\
&= j\eta_o I_o k \int \triangle(\frac{z'}{L})\sin\theta' \frac{e^{-jk|\mathbf{r}-z'\hat{z}|}}{4\pi|\mathbf{r}-z'\hat{z}|}\hat{\theta}'dz' \\
&\approx j\eta_o I_o k \underbrace{\{\int \triangle(\frac{z'}{L})dz'\}}_{L/2,\text{ area of a triangle with height 1 and base }L} \sin\theta \frac{e^{-jk|\mathbf{r}|}}{4\pi|\mathbf{r}|}\hat{\theta} = j\eta_o I_o k \frac{L}{2}\sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\theta},
\end{aligned}$$

where the condition $L \ll \lambda$ is used to justify the replacement of $|\mathbf{r} - z'\hat{z}|$ by $|\mathbf{r}| = r$.

- Notice that the result

$$\tilde{E}(\mathbf{r}) = j\eta_o I_o k \frac{L}{2} \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\theta}$$

is identical with the radiation field of the Hertzian dipole except that

  – infinitesimal length Δz has been replaced by a finite length $\frac{L}{2}$ corresponding to the **dipole half-length**.

- The corresponding radiation magnetic field of the short dipole is

$$\tilde{H}(\mathbf{r}) = jI_o k \frac{L}{2} \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\phi}.$$

  – Dipole half-length $\frac{L}{2}$ is also known as **effective length** of the short dipole antenna.

- The term *effective length* is used more broadly to denote

$$\ell(\theta) \equiv \int \frac{\tilde{I}(z)}{I_o} e^{jkz\cos\theta} dz$$

defined for any length dipole antenna having a phasor current distribution $\tilde{I}(z)$ and a phasor current I_o at the input port (or input terminals).

Radiation fields of the short dipole:

$$\tilde{\mathbf{E}} = j\eta_o I_o k \frac{L}{2} \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\theta}$$

and

$$\tilde{\mathbf{H}} = jI_o k \frac{L}{2} \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\phi}.$$

For a **short dipole** with

$$\tilde{I}(z) = I_o \triangle(\frac{z}{L}),$$

where $L \ll \lambda$, this definition yields $\ell(\theta) = \frac{L}{2}$.

For a **half-wave dipole** with

$$\tilde{I}(z) = I_o \mathrm{rect}(\frac{z}{L}) \cos(kz),$$

where $L = \frac{\lambda}{2}$, this definition yields

$$\ell(\theta) = \frac{\lambda}{\pi} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta},$$

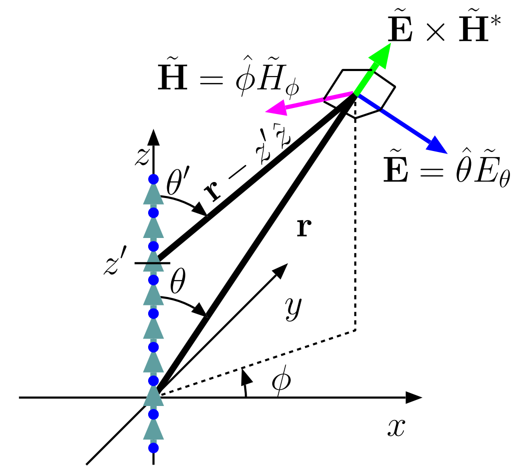
as will be shown in ECE 454.

Effective length

$$\ell(\theta) \equiv \int \frac{\tilde{I}(z)}{I_o} e^{jkz\cos\theta} dz.$$

Radiation fields of all linearly polarized antennas can be obtained from those of the Hertzian dipole by replacing "Δz" with an appropriate effective length "$\ell(\theta)$" as illustrated above.

- The justification of this general rule is as follows:



Replacing $\triangle(\frac{z'}{L})$ with an arbitrary $\frac{\tilde{I}(z')}{I_o}$ in the second line of the above expression for $\tilde{E}(\mathbf{r})$, we have

$$\begin{aligned} \tilde{E}(\mathbf{r}) &= j\eta_o I_o k \int \frac{\tilde{I}(z')}{I_o} \sin\theta' \frac{e^{-jk|\mathbf{r}-z'\hat{z}|}}{4\pi|\mathbf{r}-z'\hat{z}|} \hat{\theta}' dz' \\ &\approx j\eta_o I_o k \{\int \frac{\tilde{I}(z')}{I_o} e^{-jk|\mathbf{r}-z'\hat{z}|} dz'\} \sin\theta \frac{1}{4\pi|\mathbf{r}|} \hat{\theta}, \end{aligned}$$

where in the second line we have replaced all occurrences of $|\mathbf{r} - z'\hat{z}|$ by $|\mathbf{r}| = r$, except for in the complex exponential which is highly sensitive to z'.

- Replacements outside the exponential are easily justified for any finite L (large or small) so long as $r \gg L$.

- The same replacement cannot be justified in the exponential, even in $r \to \infty$ limit, because even a

  tiny difference between $|\mathbf{r} - z'\hat{z}|$ and $|\mathbf{r}| = r$

  would produce a

  large difference between the "angles" of $e^{-jk|\mathbf{r}-z'\hat{z}|}$ and e^{-jkr}

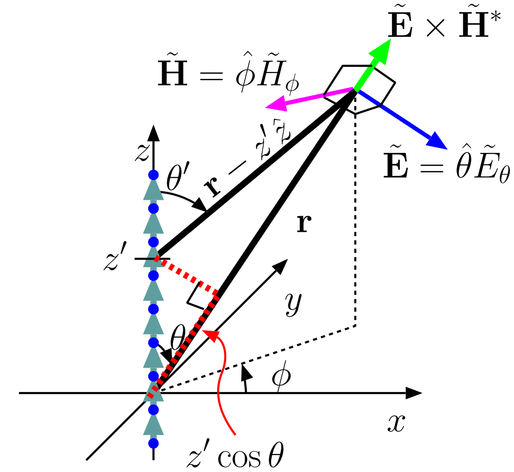
  that would matter if k were sufficiently large (or $\lambda = \frac{2\pi}{k}$ small, say compared to $\sim L$).

- In $r \to \infty$ limit, the vectors $\mathbf{r} - z'\hat{z}$ and $\mathbf{r}$ become parallel to one another, in which case it can be easily seen that (see margin)

$$\lim_{r\to\infty} |\mathbf{r} - z'\hat{z}| = r - z' \cos\theta.$$

  This *exact result* in $r \to \infty$ limit furnishes us, for finite r, with the so-called **paraxial approximation**

$$e^{-jk|\mathbf{r}-z'\hat{z}|} \approx e^{-jk(r-z'\cos\theta)} = e^{-jkr} e^{jkz'\cos\theta},$$



leading, in turn, to

$$\tilde{E}(\mathbf{r}) \approx j\eta_o I_o k \underbrace{\int \frac{\tilde{I}(z')}{I_o} e^{jkz'\cos\theta} dz'}_{\equiv \ell(\theta)} \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\theta},$$

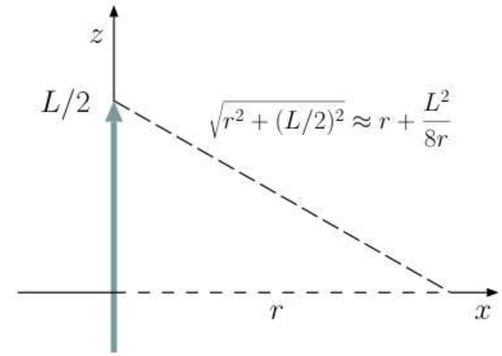
a general radiation field expression formulated in terms of effective length $\ell(\theta)$.

– This result is certainly valid for all $r \gg L$ where it makes sense to consider $\mathbf{r} - z'\hat{z}$ and $\mathbf{r}$ to be parallel vectors.

– The validity limit of paraxial approximation can be investigated more carefully by expanding $k|\mathbf{r} - z'\hat{z}|$ to a higher order, and finding under what condition high-order correction factors are really unnecessary — that exercise (see ECE 454) shows that paraxial approximation is well justified for

$$r \gtrsim \frac{2L^2}{\lambda},$$

where the "threshold distance" $2L^2/\lambda$ is known as **Rayleigh distance**.



In paraxial approximation the dashed lines are treated as having equal lengths (which is only accurate for r going to infinity), leading to a phase error of

$$\frac{kL^2}{8r} = \frac{\pi}{8}\frac{2L^2}{\lambda r} \text{ rad.}$$

The phase error is less than

$\frac{\pi}{8}$ rad and tolerable if $r > \frac{2L^2}{\lambda}$.

- Even though we have developed a general representation for the radiation fields of arbitrary dipoles in this lecture, our discussions over the next few lectures will focus mainly on short dipoles as our basic radiation elements.