12 Interference, antenna arrays — cont'd.

We continue with our study of interference effects and antenna arrays.

- Beam patterns of N-element antenna arrays examined last lecture were isotropic in ϕ direction — the main effect of increasing the array size Nd appeared to be *narrowing* the mainlobe of the pattern in θ direction.

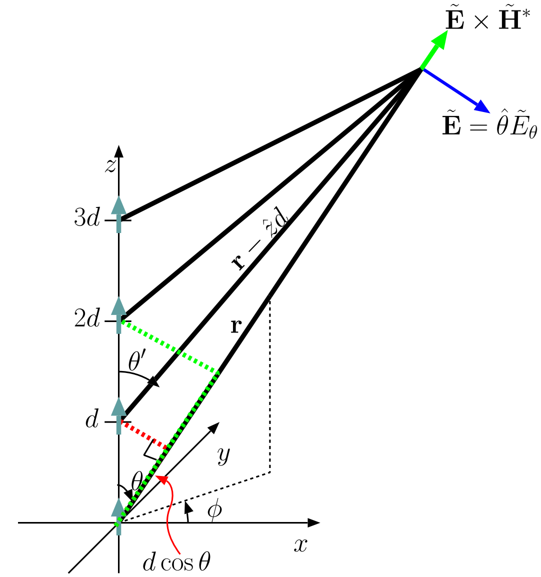
- These so-called **broadside arrays** — meaning that they mainly radiate in the "broadside direction" of the "array axis" — are good for *broadcasting purposes* at relatively high frequencies $\frac{\omega}{2\pi}$ in the FM band (∼100 MHz),

  – where array sizes Nd, in excess of many λ's, become practicable (as opposed to in AM band where $\frac{\omega}{2\pi}$ ∼1 MHz and $\lambda \sim 300$ m).

- They may also be used as "elements" of arrays built along x- or y-axis directions which we will consider next.

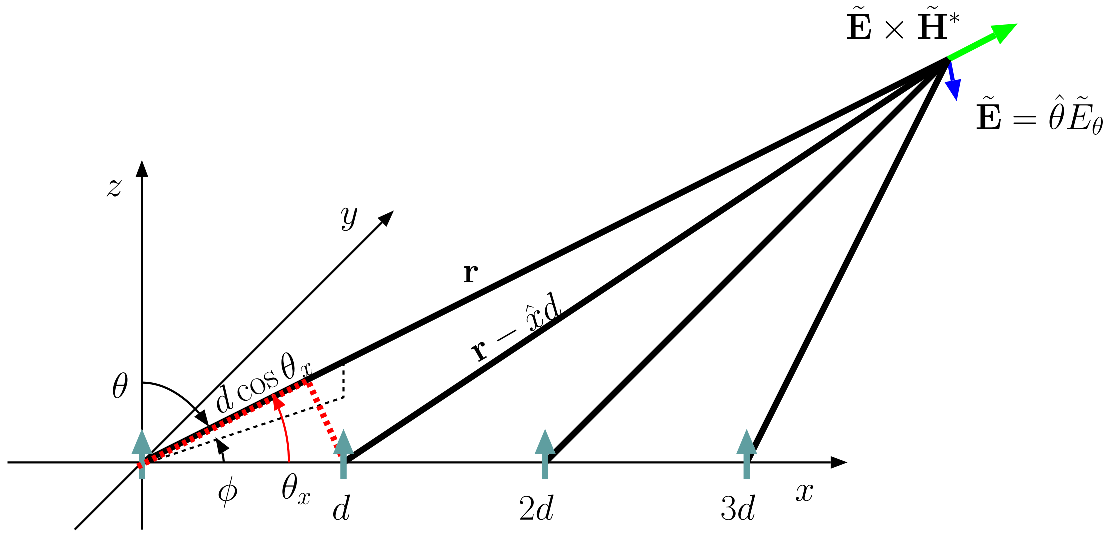
  – In that case it will be possible to produce antenna beam patterns *anisotropic* in the azimuth plane (in ϕ direction).

  – We will also consider *phasing* the element input currents so that the mainlobe of the beam can be steered into desired directions in the azimuth plane.



New vocabulary:

— **Broadside arrays**
— **Array axis**
— **Broadside direction**



- Consider an array of elements polarized in $\hat{z}$-direction positioned along the x-axis as shown above. Our initial analysis of this array will assume equal input currents I_o for all the elements. Let

  $\tilde{\mathbf{E}}_0(\mathbf{r}) \propto \dfrac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}$ denote the field at the observation point $\mathbf{r}$ due to the element at the origin.

  - Then, using the paraxial approximation, the field phasor at a distant observation point due to the next element at $(d, 0, 0)$ can be expressed in terms of $\tilde{\mathbf{E}}_0(\mathbf{r})$ as

    $$\tilde{\mathbf{E}}_1(\mathbf{r}) \approx \tilde{\mathbf{E}}_0(\mathbf{r})e^{jkd\cos\theta_x}$$

    where θ_x is the angle between vectors $\mathbf{r}$ and $\hat{x}$, i.e.,

    $\cos\theta_x = \hat{r}\cdot\hat{x} = (\sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z})\cdot\hat{x} = \sin\theta\cos\phi,$

    known as a **direction cosine**.

– Likewise,

$$\tilde{\mathbf{E}}_2(\mathbf{r}) \approx \tilde{\mathbf{E}}_0(\mathbf{r}) e^{j2kd\cos\theta_x}, \text{ etc., so that,}$$

– For an N-element array,

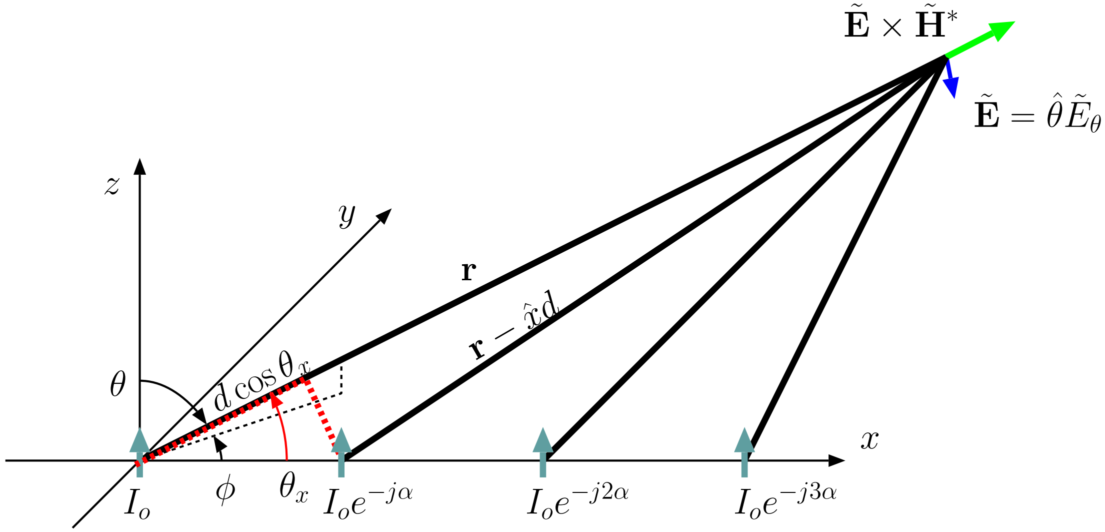
$$\tilde{\mathbf{E}}(\mathbf{r}) = \tilde{\mathbf{E}}_0(\mathbf{r})[1 + e^{jkd\cos\theta_x} + e^{j2kd\cos\theta_x} + \cdots + e^{j(N-1)kd\cos\theta_x}].$$

- The field expression above is identical in essence with the field expression for the N-element array examined in the last lecture except for the replacement of $\cos\theta$ by $\cos\theta_x$. Therefore, assuming that $\tilde{\mathbf{E}}_0(\mathbf{r})$ is due a short dipole (so that $\ell = \frac{L}{2}$ is independent of direction), we obtain the gain function for our new array by exchanging $\cos\theta$ by $\cos\theta_x$ in the gain expression obtained in the last lecture — by that procedure we arrive at

$$\begin{aligned} G(\theta,\phi) &= D\sin^2\theta \frac{\sin^2(\frac{N}{2}kd\cos\theta_x)}{N^2\sin^2(\frac{1}{2}kd\cos\theta_x)} \\ &= D\sin^2\theta \frac{\sin^2(\frac{N}{2}kd\sin\theta\cos\phi)}{N^2\sin^2(\frac{1}{2}kd\sin\theta\cos\phi)}. \end{aligned}$$

We have *at last* obtained a gain function that does actually depend on both θ and ϕ.

– Even more complicated gain expressions would be obtained if the array elements were themselves arrays (like those examined last lecture) having angle dependent ℓ's! (see HW problems).



Phased array:

- Next let's examine what happens when the element input currents are not identical, but follow a progressive phase pattern with

$$I_n = I_o e^{-jn\alpha} \text{ for } n\text{-th element located at } (nd, 0, 0)$$

where α is a phasing increment specified in radians.

In that case — since $\tilde{\mathbf{E}}_n(\mathbf{r}) \propto I_n$ — we would have a **phased array** with element field phasors

$$\tilde{\mathbf{E}}_1(\mathbf{r}) \approx \tilde{\mathbf{E}}_0(\mathbf{r})e^{j(kd\cos\theta_x - \alpha)}, \;\; \tilde{\mathbf{E}}_2(\mathbf{r}) \approx \tilde{\mathbf{E}}_0(\mathbf{r})e^{j2(kd\cos\theta_x - \alpha)}, \;\; \text{etc.},$$

and an array gain function (again, assuming short-dipole elements)

$$G(\theta, \phi) = K \sin^2\theta \frac{\sin^2(\frac{N}{2}(kd\sin\theta\cos\phi - \alpha))}{N^2\sin^2(\frac{1}{2}(kd\sin\theta\cos\phi - \alpha))}$$

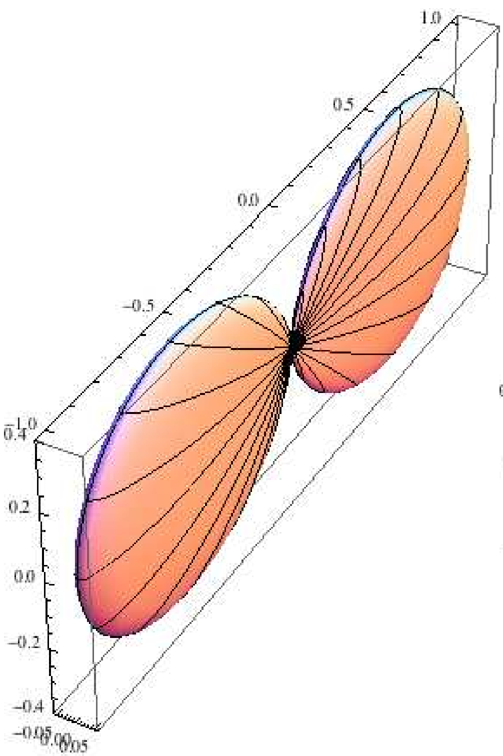
where constant K is to be determined by requiring $\int G d\Omega = 4\pi$.

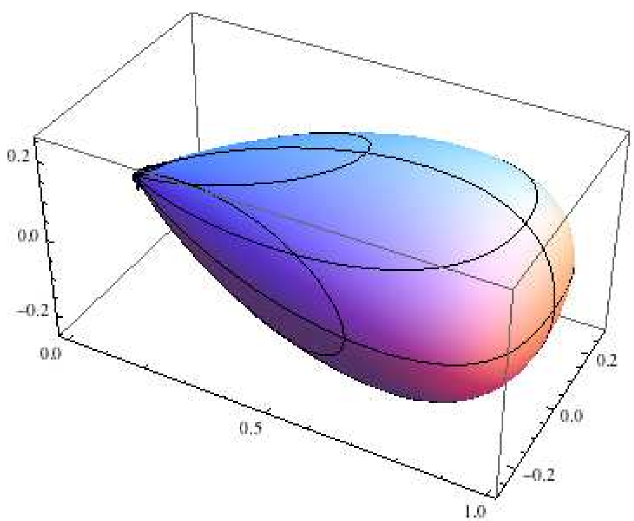
- On $\theta = 90°$ plane we have

$$G(90°, \phi) = K \frac{\sin^2(\frac{N}{2}(kd\cos\phi - \alpha))}{N^2 \sin^2(\frac{1}{2}(kd\cos\phi - \alpha))}$$

which leads to azimuth plane patterns (G/K) shown in the margin shown for $d = \frac{\lambda}{4}$, $N = 16$, and $\alpha = 0$, $\frac{\pi}{4}$, and $\frac{\pi}{2}$ radians.

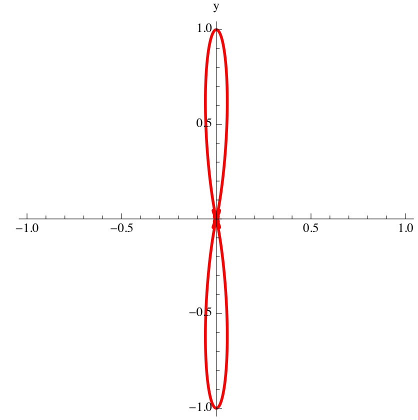
- Also, 3D plots of $G(\theta, \phi)/K$ for $d = \frac{\lambda}{4}$, $N = 16$, and $\alpha = 0$ and $\frac{\pi}{2}$ radians are shown below:



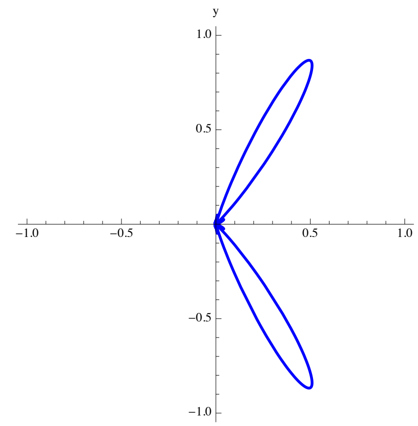


Question: The broadside pattern shown on the left is clearly a "fan beam" — how would you make the "fans" narrower (in order to increase D) in θ direction?

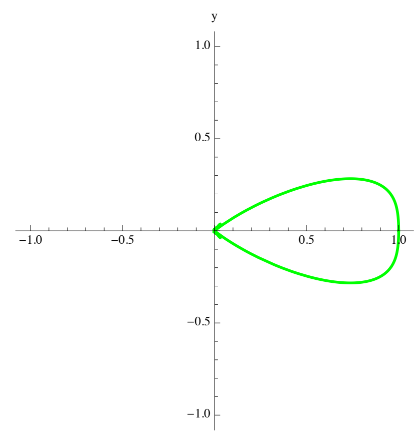
$\alpha = 0$**: Broadside array**



$\alpha = \frac{\pi}{4}$**: Steered beam:**



$\alpha = \frac{\pi}{2}$**: End-fire array**



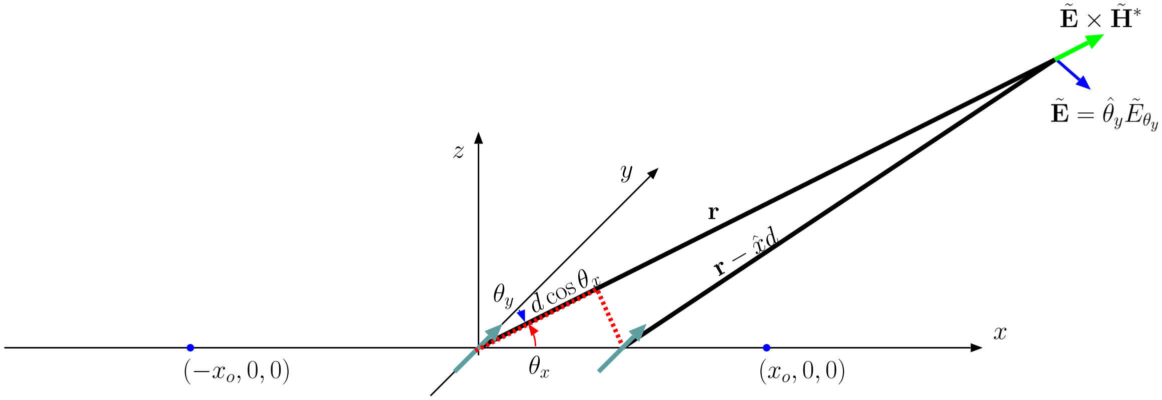
- Our discussion so far have focused on 1D arrays.
  - One class of 2D arrays consist of 1D arrays having other 1D arrays as their elements, in which case their gain functions can be formulated after multiplying the array factors of two 1D arrays and the effective length of the smallest element of the arrays.
  - We have shown examples illustrating how antenna beams with relatively small solid angles Ω_o and directivities $D = \frac{4\pi}{\Omega_o}$ can be generated.
  - We have shown how phasing can be used to steer the antenna beam patterns.

- Interference effects which are fundamental to antenna array design are mainly sensitive to

  1. antenna locations, and
  2. phases of antenna input currents.

  The next example examines these parameters in more detail.

Example 1: We have two identical $\hat{y}$-polarized short dipoles. We want to "place" them and "phase" their input currents in such a way that no power is radiated in $+x$ direction and there is a gain maximum in $-x$ direction. Determine the required positions of the dipoles and the relative phases of their input currents.



Solution: Let's place the two $\hat{y}$-polarized short dipoles at $(0,0,0)$ and $(d,0,0)$ as shown above and drive them with input currents I_1 and I_2, respectively. For $I_2 = I_1 e^{-j\alpha}$, the field phasors of the dipoles at an observation point $(x_o, 0, 0)$, $x_o \gg d$, will vary as

$$\tilde{\mathbf{E}}_1 \propto e^{-jkx_o} \quad \text{and} \quad \tilde{\mathbf{E}}_2 \propto e^{-j\alpha} e^{-jk(x_o - d)} = e^{-jkx_o} e^{j(kd-\alpha)}$$

having identical proportionality constants. Likewise, at an observation point $(-x_o, 0, 0)$, $x_o \gg d$, we will have field phasors

$$\tilde{\mathbf{E}}_1 \propto e^{-jkx_o} \quad \text{and} \quad \tilde{\mathbf{E}}_2 \propto e^{-j\alpha} e^{-jk(x_o + d)} = e^{-jkx_o} e^{-j(kd+\alpha)}.$$

Now, in order to have *destructive* interference between $\tilde{\mathbf{E}}_1$ and $\tilde{\mathbf{E}}_2$ at $(x_o, 0, 0)$ we need to have

$$e^{j(kd-\alpha)} = -1 \quad \Rightarrow \quad kd - \alpha = \pi.$$

Also to have *constructive* interference between $\tilde{\mathbf{E}}_1$ and $\tilde{\mathbf{E}}_2$ at $(-x_o, 0, 0)$ we need to have

$$e^{-j(kd+\alpha)} = 1 \quad \Rightarrow \quad kd + \alpha = 0.$$

Adding and subtracting these equations we find that

$$kd = \frac{\pi}{2} \quad \text{and} \quad \alpha = -\tfrac{\pi}{2}.$$

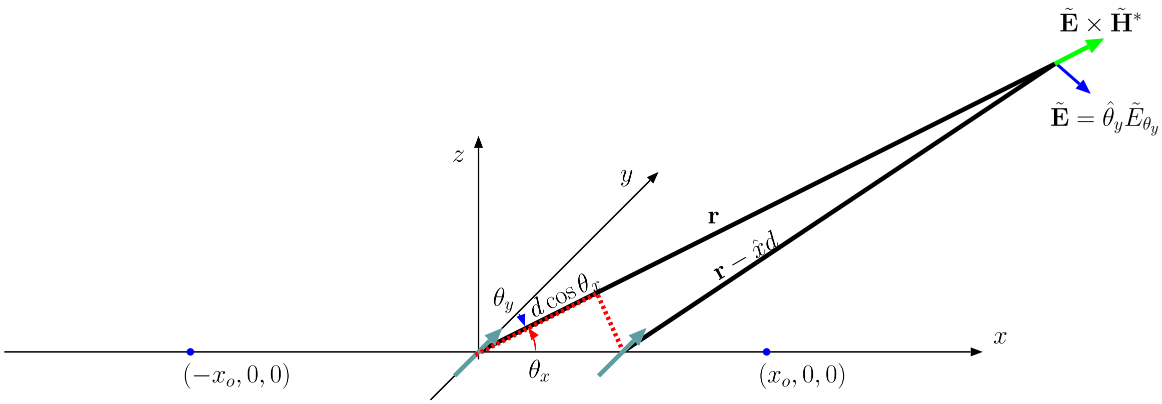
Thus

$$d = \frac{\pi}{2k} = \frac{\pi}{2\frac{2\pi}{\lambda}} = \frac{\lambda}{4}.$$

This result makes sense because, with I_1 lagging I_2 by $90°$ of phase, and $\tilde{\mathbf{E}}_1$ traveling an extra $\frac{\lambda}{4}$ compared to $\tilde{\mathbf{E}}_2$ to lose an additional phase of $90°$, $\tilde{\mathbf{E}}_1$ ends up being $180°$ out of phase with $\tilde{\mathbf{E}}_1$ at $(x_o, 0, 0)$, which is the condition for destructive interference.

Conversely, with I_2 leading I_1 by $90°$ of phase, but $\tilde{\mathbf{E}}_2$ traveling an extra $\frac{\lambda}{4}$ compared to $\tilde{\mathbf{E}}_1$ to lose that phase lead of $90°$, $\tilde{\mathbf{E}}_2$ ends up being *in phase* with $\tilde{\mathbf{E}}_1$ at $(-x_o, 0, 0)$, which is the condition for constructive interference.

Example 2*: (Difficult example) Obtain the gain function $G(\theta, \phi)$ for the 2-element array examined in Example 2.



Solution: With $I_2 = I_1 e^{-j\alpha}$, $\alpha = -\frac{\pi}{2}$ and $d = \frac{\lambda}{4}$, we have in the antenna far-field (i.e., the region where paraxial approximation justified)

$$\tilde{\mathbf{E}}(\mathbf{r}) = \tilde{\mathbf{E}}_1(\mathbf{r})(1 + e^{-j\alpha} e^{jkd\cos\theta_x}) = \tilde{\mathbf{E}}_1(\mathbf{r})(1 + e^{j\frac{\pi}{2}(\cos\theta_x + 1)})$$

where

$$\tilde{\mathbf{E}}_1(\mathbf{r}) = j\eta_o I_1 k\ell \sin\theta_y \frac{e^{-jkr}}{4\pi r}\hat{\theta}_y.$$

Consequently,

$$G(\theta,\phi) = Df(\theta,\phi)$$

such that

$$f(\theta,\phi) \propto |\ell|^2 |\sin\theta_y|^2 |1 + e^{j\frac{\pi}{2}(\cos\theta_x+1)}|^2$$

and is normalized to a peak value of 1. This is compatible with

$$\begin{aligned}
G(\theta,\phi) &= D\sin^2\theta_y \frac{|1 + e^{j\frac{\pi}{2}(\cos\theta_x+1)}|^2}{4} \\
&= D\sin^2\theta_y \frac{2 + e^{j\frac{\pi}{2}(\cos\theta_x+1)} + e^{-j\frac{\pi}{2}(\cos\theta_x+1)}}{4} \\
&= D(1-\cos^2\theta_y)\frac{1+\cos(\frac{\pi}{2}(\cos\theta_x+1))}{2} \\
&= D(1-\sin^2\theta\sin^2\phi)\frac{1+\cos(\frac{\pi}{2}(\sin\theta\cos\phi+1))}{2}.
\end{aligned}$$

A 3D plot of $G(\theta,\phi)/D$ is shown in the margin.