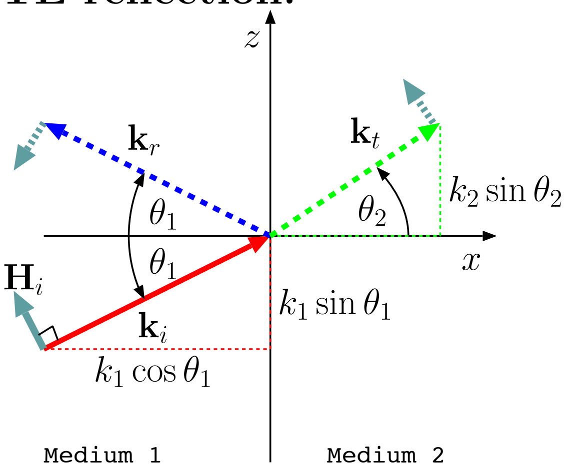
18 Reflecting plates, monopole antennas, corner reflectors

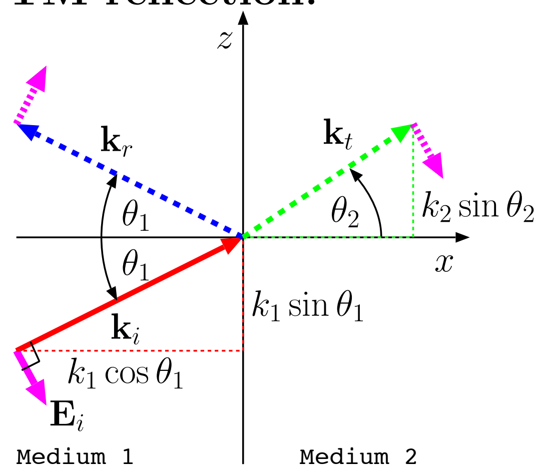
TE reflection:



$$\Gamma_\perp \equiv \frac{E_{yr}}{E_{yi}} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

$$\tau_\perp \equiv \frac{E_{yt}}{E_{yi}} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2},$$

TM reflection:



$$\Gamma_\parallel \equiv -\frac{E_r}{E_i} = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} = \frac{E_{zr}}{E_{zi}}$$

$$\tau_\parallel \equiv \frac{E_t}{E_i} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1}.$$

- In deriving the transmission and reflection rules for TE and TM modes summarized above we assumed lossless propagation media during the last two lectures.

- The equations can be easily modified as described next if either medium

1 or medium 2 or both have non-zero conductivities σ_1 and/or σ_2.

In general, in the case of a non-insulating medium with a finite conductivity σ, we expect a conduction current $\tilde{\mathbf{J}} = \sigma\tilde{\mathbf{E}}$, in which case the plane-wave form of Ampere's law can be cast as

$$\begin{aligned} -j\mathbf{k} \times \tilde{\mathbf{H}} &= \sigma\tilde{\mathbf{E}} + j\omega\epsilon\tilde{\mathbf{E}}, \\ &= j\omega(\epsilon + \frac{\sigma}{j\omega})\tilde{\mathbf{E}}. \end{aligned}$$

Since this equation differs from the non-conducting case only by having $\epsilon + \frac{\sigma}{j\omega}$ in place of ϵ, propagation parameters

$$k = \omega\sqrt{\mu\epsilon} \quad \text{and} \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

of non-conducting media are modified as

$$k = \omega\sqrt{\mu(\epsilon + \frac{\sigma}{j\omega})} \quad \text{and} \quad \eta = \sqrt{\frac{\mu}{\epsilon + \frac{\sigma}{j\omega}}},$$

respectively, in homogeneous conducting media. In other words a conducting medium is treated as a dielectric with a permittivity $\epsilon + \frac{\sigma}{j\omega}$.

- Consider the wavenumber

$$k = \omega\sqrt{\mu(\epsilon + \frac{\sigma}{j\omega})}$$

in a medium with $\epsilon \gg \sigma/\omega$. In that case — poor conductor approximation — we can approximate k as

$$\begin{aligned} k &= \omega\sqrt{\mu(\epsilon - j\frac{\sigma}{\omega})} = \omega\sqrt{\mu\epsilon(1 - j\frac{\sigma}{\omega\epsilon})} \approx \omega\sqrt{\mu\epsilon}(1 - j\frac{\sigma}{2\omega\epsilon}) \\ &= \omega\sqrt{\mu\epsilon} - j\frac{1}{2}\sqrt{\frac{\mu}{\epsilon}}\sigma \equiv k' - jk'', \end{aligned}$$

with

$$k' \equiv \mathrm{Re}\{k\} \approx \omega\sqrt{\mu\epsilon} \quad \text{Propagation constant}$$

and

$$k'' \equiv -\mathrm{Im}\{k\} \approx \frac{1}{2}\sqrt{\frac{\mu}{\epsilon}}\sigma \quad \text{Attenuation constant.}$$

These terms are applicable since

$$e^{-j\mathbf{k}\cdot\mathbf{r}} = e^{-jks} = e^{-j(k'-k'')s} = e^{-k''s}e^{-jk's}$$

clearly signify an attenuating plane-wave field with distance s measured in the direction of a unit vector $\hat{k}$ such that $\mathbf{k}$ introduced above relates to $k = k' - jk''$ as in

$$\mathbf{k} = \hat{k}(k' - jk'').$$

- Conversely, in a medium where $\epsilon \ll \sigma/\omega$ — good conductor approximation — we can approximate k as

$$\begin{aligned} k &= \omega\sqrt{\mu(\epsilon - j\frac{\sigma}{\omega})} \approx \omega\sqrt{-j\frac{\mu\sigma}{\omega}} = \sqrt{-j}\sqrt{\omega\mu\sigma} \\ &= \frac{1-j}{\sqrt{2}}\sqrt{\omega\mu\sigma} = (1-j)\sqrt{\pi f\mu\sigma} \equiv k' - jk'' \end{aligned}$$

Clearly, in this case the **penetration depth** δ (recall ECE 329 — distance for the field to decay one e-fold) is

$$\delta = \frac{1}{k''} = \frac{1}{\sqrt{\pi f \mu \sigma}},$$

and this quantity vanishes in the limit $\sigma \to \infty$ — the meaning of this is, TEM waves cannot penetrate regions of perfect electrical conductors.

Example 1: Consider a plane wave of frequency $f = 400$ MHz propagating in a conductive medium with conductivity $\sigma = 4 \times 10^7$ S/m. Given that the wave phasor is

$$\tilde{\mathbf{E}}(x) = \hat{y} E_o e^{-jkx} = \hat{y} E_o e^{-k''x} e^{-jk'x}.$$

determine $\mathbf{E}(x,t)$ and $\mathbf{H}(x,t)$ and the penetration depth (skin depth) δ as well as the propagation velocity $v_p = \omega/k'$. Assume that $\mu = \mu_o$ and $\epsilon = \epsilon_o$ in the medium.

Solution: We first note that in this case

$$\frac{\sigma}{\omega} = \frac{4 \times 10^7}{2\pi \times 400 \times 10^6} = \frac{1}{20\pi} \gg \epsilon = \epsilon_o \approx \frac{1}{36\pi \times 10^9}.$$

Thus

$$k' = k'' \approx \sqrt{\pi f \mu \sigma} = \sqrt{\pi 400 \times 10^6 \times 4\pi \times 10^{-7} \times 4 \times 10^7} = 8\pi \times 10^4 \, \frac{\text{rad}}{\text{m}}.$$

Also,

$$\eta = \sqrt{\frac{\mu}{\epsilon + \frac{\sigma}{j\omega}}} \approx \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{2\pi \times 400 \times 10^6 \times 4\pi \times 10^{-7}}{4 \times 10^7}} \angle 45^\circ = 2\pi\sqrt{2} \angle 45^\circ \text{ m}\Omega.$$

Therefore, we have

$$\mathbf{E}(x,t) = \hat{y} E_o e^{-8\pi\times 10^4 x} \cos(8\pi \times 10^8 t - 8\pi \times 10^4 x)\,\frac{\mathrm{V}}{\mathrm{m}}$$

and

$$\mathbf{H}(x,t) = \hat{z}\frac{E_o e^{-8\pi\times 10^4 x}}{2\pi\sqrt{2}} \cos(8\pi \times 10^8 t - 8\pi \times 10^4 x - 45^\circ)\,\frac{\mathrm{kA}}{\mathrm{m}}.$$

The penetration depth is

$$\delta = \frac{1}{k''} = \frac{1}{8\pi \times 10^4} = \frac{1}{80\pi} \times 10^{-3}\,\mathrm{m},$$

clearly a small fraction of a millimeter. Finally the propagation velocity is

$$v_p = \frac{\omega}{k'} = \frac{8\pi \times 10^8}{8\pi \times 10^4} = 10^4\,\frac{\mathrm{m}}{\mathrm{s}}.$$

Example 2: A plane TEM wave propagating in air with a phasor

$$\tilde{\mathbf{E}}_i(x) = \hat{y} e^{-j\frac{k_1}{\sqrt{2}}(x+z)}$$

is incident $x = 0$ plane at the planar boundary of a good conductor for which $\sigma \gg \omega\epsilon$. Determine the transmitted field phasor $\tilde{\mathbf{E}}_t(x)$.

Solution: Formally

$$\tilde{\mathbf{E}}_t(x) = \hat{y}\tau_\perp e^{-j(k_{2x}x + k_{2z}z)}$$

where, according to Snell's law,

$$k_{2z} = k_{1z} = \frac{k_1}{\sqrt{2}}$$

and

$$k_{2x} = \sqrt{k_2^2 - k_{2z}^2} \approx \sqrt{-j\mu\sigma\omega - \frac{k_1^2}{2}} \approx \sqrt{-j\mu\sigma\omega} = k' - jk''$$

with

$$k' = k'' = \sqrt{\pi f \mu \sigma}.$$

Thus

$$\tilde{\mathbf{E}}_t(x) = \hat{y}\tau_\perp e^{-k''x} e^{-j(k'x + \frac{k_1}{\sqrt{2}}z)}$$

which is a ***non-uniform plane wave*** since $\tilde{\mathbf{E}}_t$ is not a constant on planes of constant phase as a consequence of $e^{-k''x}$ factor.

The transmission coefficient $\tau_\perp$ can be computed using the usual formula for $\tau_\perp$, but with a complex valued $\cos\theta_2$ obtained from $\sin^2\theta_2 + \cos^2\theta_2 = 1$ and Snell's law, $k_2 \sin\theta_2 = k_1 \sin\theta_1$, used with complex k_2 and $\sin\theta_2$ — under the "good conductor" conditions considered here, it will be the case that $|\tau_\perp| \ll 1$.

A *plane wave* is said to be *non-uniform* if the wavevector

$$\mathbf{k} = (k_x, k_y, k_z)$$

contains a complex valued component so that the field is not constant on planes of constant phase.

- Assume that medium 2 is a perfect electrical conductor (PEC), i.e., $\sigma_2 \to \infty$. In that case

$$\eta_2 = \sqrt{\frac{\mu}{\epsilon + \frac{\sigma}{j\omega}}} = 0 \text{ (PEC is like a "short"), and}$$

TE reflection:

$$\Gamma_\perp \equiv \frac{E_{yr}}{E_{yi}} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} \to -1$$

$$\tau_\perp \equiv \frac{E_{yt}}{E_{yi}} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} \to 0$$

TM reflection:

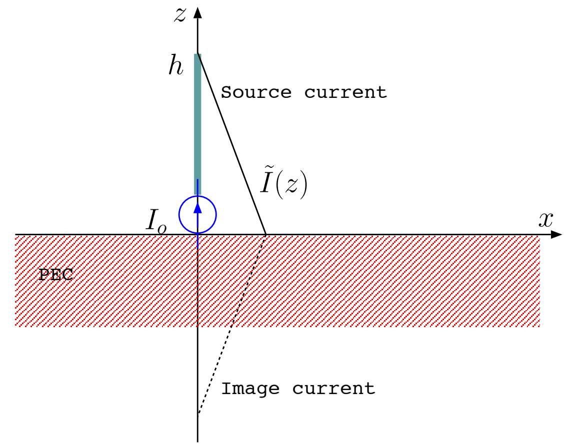
$$\Gamma_\parallel \equiv -\frac{E_r}{E_i} = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} = \frac{E_{zr}}{E_{zi}} \to -1$$

$$\tau_\parallel \equiv \frac{E_t}{E_i} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} \to 0.$$

Conclusion: all plane waves incident on a PEC boundary will reflect in such a way that tangential $\tilde{\mathbf{E}}$ at the bounding surface (this is the total field summed on the dielectric side of the boundary) is everywhere zero.

- We will next examine the consequences of this conclusion on antennas placed near conducting planes.

Monopole above reflecting surface:



- Consider a a straight wire of a length h "end-fed" by an independent current source $\tilde{I}(0) = I_o$ connected as shown in the margin between the wire and an infinite ground plane. For $h \ll \lambda = \frac{2\pi}{k}$ we may assume a current distribution

$$\tilde{I}(z) = I_o \triangle(\frac{z}{2h}) u(z)$$

that drops linearly from I_o to 0 across the length of the wire.

We next construct the radiation field of this so-called *vertical monopole* antenna by postulating that the monopole will radiate, into the half-space $z > 0$, like a short-dipole of a length $L = 2h$ having a triangular current distribution

$$\tilde{I}_d(z) = I_o \triangle(\frac{z}{2h})$$

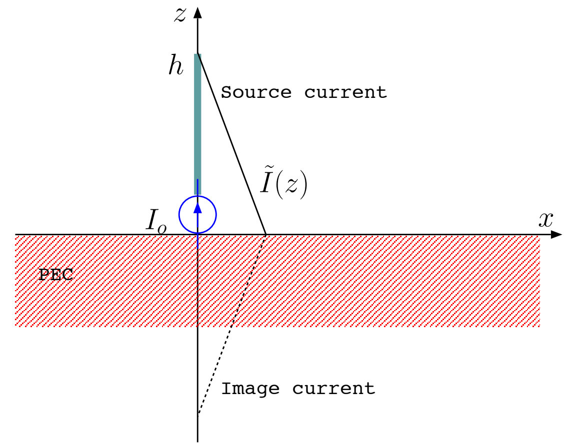
that matches $\tilde{I}(z)$ for $z > 0$ and is considered to be a "image" current of $\tilde{I}(z)$ for $z < 0$.

We justify the postulate by observing that:

1. the field generated by $\tilde{I}_d(z)$ is in $\hat{\theta} = -\hat{z}$ direction on $z = 0$ surface, and therefore it satisfies the boundary condition of having zero tangential $\tilde{\mathbf{E}}$ on the perfectly conducting surface,
2. the field generated by $\tilde{I}_d(z)$ also satisfies the boundary condition of having zero tangential $\tilde{\mathbf{E}}$ on the perfectly conducting surface of the h-long monopole wire, since the wire is just the upper half

of a two-wire dipole of length $L = 2h$ (on which the condition is satisfied *ipso facto*),

3. Maxwell's equations (ME) necessarily have a *unique* solution for each possible configuration of boundary conditions (BC) — *a* solution of ME's matching the given BC is *the* solution!



In view of above, the radiation field of the monopole is

$$\tilde{\mathbf{E}} = \begin{cases} j\eta_o I_o k \frac{L}{2} \sin\theta \frac{e^{-jkr}}{4\pi r}\hat{\theta} & \text{for } z > 0 \\ 0 & \text{for } z < 0, \end{cases}$$

where $\frac{L}{2}$ should be replaced by h. As this result indicates, a monopole radiates its entire power to one hemisphere as opposed to a dipole in free space radiating equally into two hemispheres.

- In the above description of the radiation of the monopole, the bottom half of the current distribution $\tilde{I}_d(z)$ is said to be the "image" of the source current $\tilde{I}(z)$ on the monopole. The image is really "imaginary" in the sense that the "real", actual, sources of the radiated field $\tilde{\mathbf{E}}$ in the upper hemisphere are

  1. $\tilde{I}(z)$ on the monopole, and
  2. a surface current $\tilde{\mathbf{J}}_s(x, y)$ induced on $z = 0$ surface in order to satisfy the BC $\hat{z} \times \tilde{\mathbf{H}}(x, y, 0) = \tilde{\mathbf{J}}_s(x, y)$.

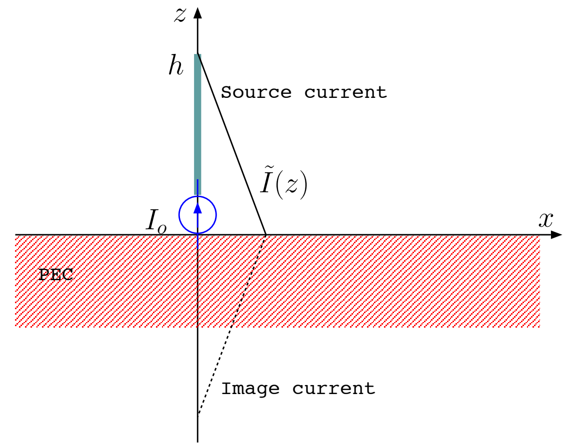
- A *short* monopole of length $h \ll \lambda$ radiates half as much power as a short-dipole of length $L = 2h$ having an equal input current I_o. Therefore R_{rad} for monopole is half of

$$R_{rad} = 20\pi^2(\frac{L}{\lambda})^2 = 20\pi^2(\frac{2h}{\lambda})^2$$

of the dipole, i.e.,

$$R_{rad,mono} = 10\pi^2(\frac{2h}{\lambda})^2 = 40\pi^2(\frac{h}{\lambda})^2$$

in ohms.



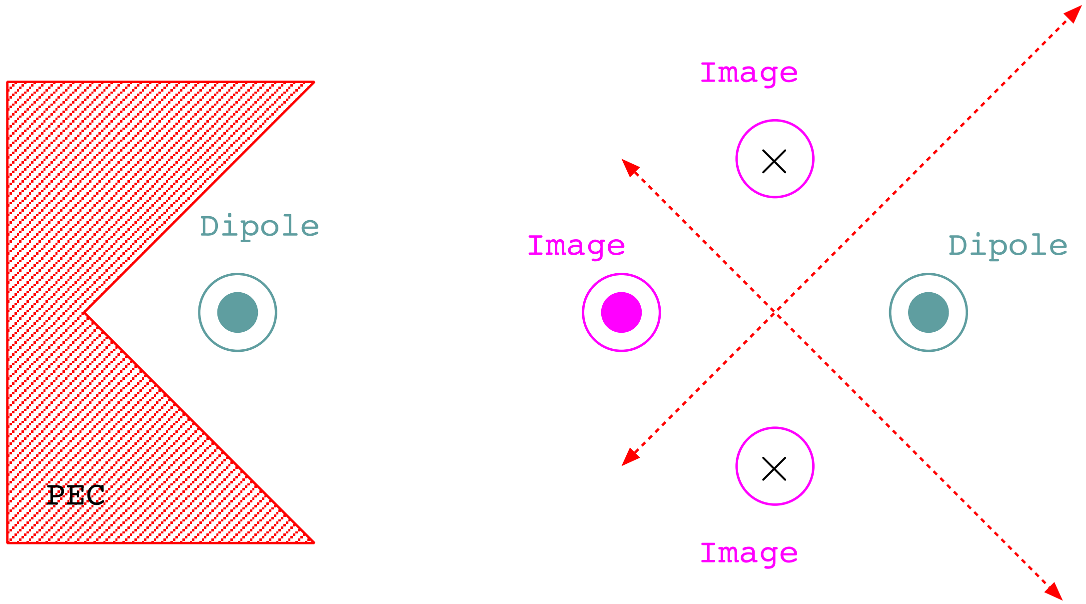
- Directivity of monopole is twice the directivity of short-dipole, i.e., $D = 3$, since the beam solid angle Ω_o of the monopole is half the solid angle of the dipole (why?).

- Finally, a monopole of length $h = \frac{\lambda}{4}$ is called a *quarter-wave monopole.*

  – In analogy with a half-wave dipole, the quarter-wave monopole has a radiation resistance of about 36 ohms and a directivity of 3.28.

Corner reflector antenna

- The following diagram depicts a "corner reflector" antenna on the left, and its image based model as a 4-element array.



(a) Corner reflector (b) Image based model

  - Note that the image elements in the model have been so selected that the 4-element array has tangential electric field nulls along the conducting walls of the corner reflector placed next to the $\hat{z}$-polarized dipole antenna (seen from the top) shown on the left.
  - The field of the corner reflector antenna matches the field of the 4-element array in the rightmost quadrant in the diagram bounded by the diagonal lines. The field can be calculated easily by using an array factor that depends on the distance of the dipole from the reflecting corner (see HW).